Glossary

A

absolute comparison An absolute comparison is an additive comparison between quantities. In an absolute comparison, 7 out of 10 is considered to be larger than 4 out of 5, since 7 is larger than 4.

algebraic ideas Algebraic ideas include patterns, variables, and functions, and are the building blocks of algebraic thinking.

algebraic structure Algebraic structures are systems with objects and operations, and the rules or properties governing those operations, that can be used to calculate and solve equations.

algorithm An algorithm is a recipe or a description of a mechanical set of steps for performing some task.

amplitude The amplitude of a cyclic function measures the height of the cyclic function, relative to its average. The amplitude can be determined by the formula: amplitude = (high – low) / 2.

associativity Associativity is a property of some sets and binary operations where the order in which the operation is carried out may be manipulated to produce the same results. Addition is associative: (2 + 3) + 4 is equivalent to 2 + (3 + 4). Subtraction is not associative: 2 – (3 – 4) is not equivalent to (2 – 3) – 4.

axes The axes of a graph are the base (or zero) values of two quantities that are being compared in a coordinate graph. The horizontal axis is often referred to as the *x*-axis and the vertical axis is often referred to as the *y*-axis.

B

backtracking Backtracking is a method of solving equations that involves "undoing" the operations in an equation to work backward from an output to an input.

base In the exponential equation $8 = 2^3$, 2 is the base and 3 is the exponent.

binary operation A binary operation is an operation that combines two objects of one type to form another object of the same type. For example, in the calculation 2 + 3 = 5, "+" is a binary operation. Addition, subtraction, multiplication, division, and exponentiation are all binary operations, but there are many others at the heart of the study of algebraic structures.

C

closed-form description A closed-form description of a pattern tells how to get from any input to its output, without having to know any previous outputs. A rule such as "take the input, triple it, and add two" is a closed-form description of a pattern.

closure A property of some sets and binary operations, closure means that the output of a binary operation is always within the set of the inputs. Addition is closed for the set of even numbers: 4 + 6 = 10; any two even numbers will add up to an even number. Addition is *not* closed for the set of odd numbers: 3 + 5 = 8, and 8 is not an odd number.

common difference A table of common differences is formed by finding the difference between successive outputs. A table of second common differences can be formed by finding the difference between the differences. Common difference can be used to determine if a table of values comes from a linear or quadratic function. See Session 7, Part D for more information.

commutativity A property of some sets and binary operations, commutativity means that the order of the objects is not important. Addition is commutative: 2 + 3 is equivalent to 3 + 2. Subtraction is not commutative: 2 – 3 is not equivalent to 3 – 2.

covering up Covering up is a method of solving equations by covering up a portion of the equation to yield a simpler, one-step equation. For example, if $6 + 2m = 12$, cover up "$2m$" to give the equation $6 + ? = 12$. Because $6 + 6 = 12$, the "?" must be 6, so $2m = 6$ and $m = 3$.

Glossary, cont'd.

cyclic function A cyclic function is a function whose outputs repeat in a cycle. A traffic light is an example of a cyclic function. Cyclic functions have important applications in astronomy and the study of wave motion, including sound and light waves.

D

dependent variable In a function where the value of variable *A* depends on the value of variable *B*, variable *A* is referred to as the dependent variable and variable *B* is referred to as the independent variable. In a table with inputs and outputs, the input is the independent variable and the output is the dependent variable.

direct variation A direct variation is a relationship between inputs and outputs in which the ratio of inputs and outputs is always the same.

distributivity Distributivity is a property of some sets and two binary operations, such as addition and multiplication. For these operations, $a(b + c) = (ab) + (ac)$.

E

equivalence Equivalence is a relationship between two quantities, usually represented by the equal sign (=). There are three important properties of equivalence:

1. Every quantity is equivalent to itself ($3 = 3$)
2. Equivalency does not depend on the sides of the equation (if $2 + 3 = 5$, then $5 = 2 + 3$)
3. With three quantities *a*, *b*, and *c*, if $a = b$ and $b = c$, then $a = c$ (if $6 + 9 = 15$ and $15 = 5 \times 3$, then $6 + 9 = 5 \times 3$).

exponent In the exponential equation $8 = 2^3$, 2 is the base and 3 is the exponent.

exponential function In an exponential function, the independent variable is an exponent in an equation. Functions like $y = 2^x$ and $y = 10(.5)^x$ are exponential functions. An exponential function has a constant ratio between successive outputs. For example, in $y = 2^x$, each time *x* grows by 1, *y* is multiplied by 2.

exponential decay function An exponential decay function is an exponential function that decreases. The function $y = (.5)^x$ is an exponential decay function; as *x* increases, *y* decreases. The decay of nuclear waste is an example of exponential decay.

exponential growth function An exponential growth function is an exponential function that increases. The function $y = 3^x$ is an exponential growth function; as *x* increases, *y* increases. Population growth and investment growing with interest are examples of exponential growth.

F

factors The factors of a whole number are the numbers that divide evenly into it. For example, the factors of 12 are 1, 2, 3, 4, 6, and 12.

false position False position is a method of solving equations by assuming a convenient number is the solution, then adjusting that number to find the actual solution.

field A set *S* and two operations form a field if three conditions are met: the set is a group under the first operation with commutativity, the set is a group under the second operation when the identity of the first operation is removed, and distributivity is satisfied. All fields share many basic properties, including the ability to solve linear equations. Examples of fields include the real numbers under addition and multiplication, and the integers mod 7 (see *modular arithmetic*) under addition and multiplication.

figurate number A figurate number is a number found by counting the items in a regular geometric shape. The most common figurate numbers are square numbers and triangular numbers. For example, 9 is a square number because it can be formed by the items in a 3×3 square. The number 10 is a triangular number because it can be formed by the items in a triangle with 4 items per side (such as the arrangement of bowling pins). Figurate numbers can be formed from a variety of geometric shapes, which can include pentagons, hexagons, cubes, and pyramids.

Glossary, cont'd.

function A function is any relationship between inputs and outputs in which each input leads to exactly one output. It is possible for a function to have more than one input that yields the same output.

function machine A function machine is a way of visualizing functions and their inputs and outputs.

G

group A set *S* and operation * form a group if the set and operation have associativity, closure, identity, and inverses for every element in *S*. Examples of groups include the integers under addition, the positive real numbers under multiplication, and the rational numbers under addition.

independent variable In a function where the value of variable *A* depends on the value of variable *B*, variable *A* is referred to as the dependent variable and variable *B* is referred to as the independent variable. In a table with inputs and outputs, the input is the independent variable and the output is the dependent variable.

I

identity element The identity element is an element that has no effect on other elements under a binary operation. In addition with the set of real numbers, zero is the identity element: $7 + 0 = 7$, $0 + 7 = 7$, and for *any* number a, $a + 0 = a$. In multiplication, 1 is the identity element: $5 \times 1 = 5$, $1 \times 5 = 5$, and for *any* a, $a \times 1 = a$.

input Almost all rules and algorithms start with an input and end with an output. For example, if a rule says to add 3 to a number, the input of 8 leads to the output of 11.

intercept An intercept is an intersection of a graph with one of the axes. An intersection with the horizontal axis is often referred to as an *x*-intercept, and an intersection with the vertical axis is often referred to as a *y*-intercept.

integer An integer is any number which is a counting number, zero, or the opposite of a counting number. The set of integers is sometimes written as $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$.

intersection An intersection of two graphs is a point (or points) shared by both graphs.

inverse The inverse of an element a in a binary operation is the element b, which produces the identity for that operation. In addition with the set of real numbers, zero is the identity, so the inverse of 3 is (-3). This is true because $3 + (-3) = 0$, and zero is the identity. In multiplication with the set of real numbers, 1 is the identity. Therefore, the inverse of 5 is (1/5), because $5 \times (1/5) = 1$.

inverse proportion An inverse proportion is a function in which the output changes in a reciprocal relationship to the input. In other words, if the input doubles, the output is cut in half.

inverse variation An inverse variation is a relationship between two variables in which a change in one variable results in an inverse, or reciprocal, change in the other. If one variable is multiplied by 3, the other is divided by 3. Forms of inverse variation are $A \times B = k$ or $B = k / A$, where k is a constant number and A and B are the two variables.

iterate To iterate a function means to repeat its algorithm, using the previous output as the next input.

L

linear relationship A constant rate produces a linear relationship between two variables.

M

mathematical thinking tools Mathematical thinking tools are analytic habits of mind, like reasoning skills, problem-solving skills, and the ability to represent a situation in different forms.

modular arithmetic In modular arithmetic, the result of a calculation is the remainder when dividing by the modulus. For example, 33 mod 7 would give the remainder when 33 is divided by 7; 33 divides evenly into 7 four times, leaving a remainder of 5. Therefore, $33 \bmod 7 = 5$. An alternate form is to write $33 \equiv 5 \pmod 7$. Modular arithmetic is sometimes referred to as remainder arithmetic, since its calculations are done with remainders.

Glossary, cont'd.

N

network A network of function machines is made when two or more function machines are connected so that the output from one function machine becomes the input for the next.

O

origin The origin of a coordinate system is the point (0, 0).

output Almost all rules and algorithms start with an input and end with an output. For example, if a rule says to add 3 to a number, the input of 8 leads to the output of 11.

P

parabola The graph of a quadratic function is called a parabola.

period The period of a cyclic function measures how long it takes to complete one cycle of the function. For example, the period of time it takes for Earth to revolve around the sun is one year.

polygon A polygon is a two-dimensional figure made up of sides of any length. A triangle, rectangle, pentagon, and octagon are all polygons. Note that the sides of a polygon do not all have to be the same length.

prime number A prime number is a whole number with exactly two factors: 1 and the number itself. For example, the numbers 2, 7, 13, 19, and 31 are prime. The numbers 0, 1, 12, and 100 are not prime. In particular, 1 is not a prime because it does not have exactly two factors (1 is the only factor).

proportional relationship A proportional relationship is a relationship between inputs and outputs in which the ratio of inputs to outputs is always the same.

Q

quadratic function A quadratic function is a function in which the independent variable is squared. The function $y = x^2$ is the most basic quadratic function. All quadratic functions fit the form $y = Ax^2 + Bx + C$, where A, B, and C can be any real number (although A cannot be zero).

R

rate A rate describes how much one variable changes in relation to another.

recursive description A recursive description of a pattern tells you how to proceed from one step to the next. For example, a recursive description might be, "Add 2 to the value of the output each time the input goes up by 1." The Fibonacci sequence, in which each output is the sum of the two numbers before it, is a recursive description of a pattern.

relative comparison A relative comparison is a multiplicative, or proportional, comparison between quantities. In a relative comparison, 4 out of 5 is considered to be larger than 7 out of 10, since 4/5 is larger than 7/10.

representation A representation of a problem may be a table, a graph, an equation, or a written rule (in words or symbols).

S

set A set is a list of items, or elements, placed in brackets, for example, {2, 3, 4} or {red, blue, yellow}. Sets may contain an unlimited number of elements: {0, 1, 2, 3, ...}. The set containing no elements, { }, is called the empty set or null set.

slope The slope of a line is often described as the ratio of rise to run. Slope is also the amount that the dependent variable changes for each increase by 1 in the independent variable. The formula for slope is: slope = (change in y) / (change in x).

Glossary, cont'd.

solution set The solution set to an equation is the set of all values for the variables in the equation that make the equation true. Here are some examples:

1. The equation $x + 5 = 9$ has the solution set {4}, since $x = 4$ is the only way to make the equation true.
2. The equation $x^2 = 9$ has the solution set {3, -3}. This represents the two different numbers that x can be.
3. The equation $x + 3 = x + 4$ has the solution set { }. This means that there are no values for x which make the equation true. The set { } is referred to as the null set, or empty set.
4. The equation $2x + y = 10$ has an infinite number of solutions, including (5, 0), (4, 2), and many other ordered pairs of values. The solution set would be written as {(5, 0), (4, 2), ...}.

system of equations A system of equations is a set of two or more equations with a common solution. Systems of equations may have multiple variables and multiple solutions. If each equation of a system is graphed, the solutions will be any points where all the graphs intersect.

V

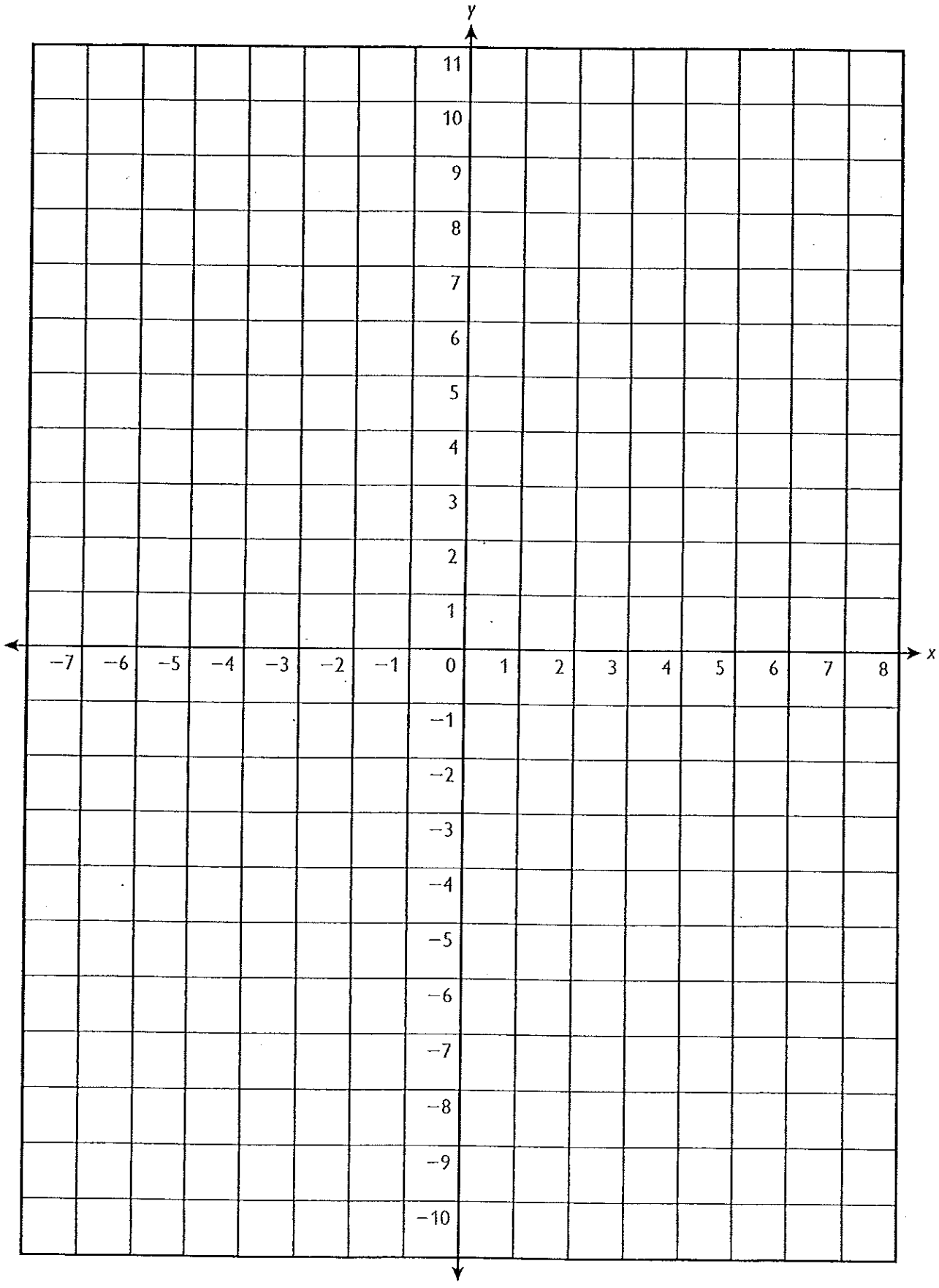
variable The term variable can have different meanings:

1. An indeterminate, a symbol used to represent generalized arithmetic (such as $0 \times a = 0$)
2. An unknown, used to represent a particular number (such as $a + 3 = 7$)
3. A relationship between quantities (such as $a + b = 10$)
4. A term used to help in understanding the study of algebraic structures (such as $a + b = b + a$)

W

whole number A whole number is a counting number or zero. The whole numbers can be written as the series 0, 1, 2, 3, ... Negative numbers and fractions are not included in the set of whole numbers.

Coordinate Plane



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