in Brief

K-12 Mathematics & Science
TEACHING CONSIDERATIONS
Elementary Students Can Reason Algebraically

The research summarized in this *in Brief* showed that elementary students can learn to think about arithmetic in ways that both enhance their learning of arithmetic and provide a foundation for learning algebra. If you want to apply these ideas in your classroom, consider the following:

Build a Foundation for Learning Algebra

- **Ask questions** that provide a window into children's understanding of important mathematical ideas. For example, students' responses to the number sentence "9 + 6 = □ + 8" tells a great deal about their understanding of the meaning of the equal sign. Probe students' reasons for their answers. Ask students why they answered as they did.
- **Provide students opportunity to discuss** and resolve different conceptions of mathematical ideas. For example, the different conceptions of the equal sign that emerge from students' solutions to the open number sentence "9 + 6 = □ + 8" can provide the basis for a productive discussion.
- **Provide students with equations** that help them understand that the equal sign represents a relation between numbers, not a signal to carry out the preceding calculation. Examples include "□ = 8 + 9," "8 + 6 = 6 + □," "9 + 6 = □ + 8." Vary the format of number sentences. Include sentences in which the answer does not come right after the equal sign.
- **Provide students with true and false number sentences** that challenge their misconceptions about the equal sign (e.g., 8 = 5 + 3, 9 = 9, 7 – 4 = 7 – 4).
- **Provide students problems that encourage them to make generalizations** about basic number properties (see "Number Sentences to Elicit Generalizations".) When they provide an answer to one of the problems, ask them how they know their answer is correct. That often will result in their stating a generalization such as "When you subtract a number from itself, you get zero." When they do state a generalization like this, ask, for example, "Is that true for all numbers?"
- **Have students justify generalizations** they or their peers propose (see page 3 for an example). Justification of generalizations requires more than providing a lot of examples (e.g., 8 × 5 = 5 × 8). By expecting children to justify their claims, you can help them gain skills in presenting mathematical arguments and proofs. Use the questions "Will that be true for all numbers?" and "How do you know that is true for all numbers?" repeatedly to encourage students to recognize that they need to justify their claims in mathematics.

NUMBER SENTENCES
to Elicit Generalizations

Is this number sentence true or false?

97 – 97 = 0	48 × 0 = 48 *
37 + 58 = 58 + 37	56 ÷ 0 = 0 *

What number can you put in the box to make this a true sentence?

□ + 74 = 74 35 × □ = 8 × 35

* denotes a false number sentence

Form a Teaching Community Focused on Students' Mathematical Thinking

- **Make classrooms a place where both you and your students are learning.** Engaging in regular inquiry about students' mathematical thinking can be one of the most powerful forms of professional development. Such inquiry does not, however, thrive in isolation. Seek out other teachers who share your interest in talking about students' thinking and share with one another the interesting observations you are making about students' mathematical thinking in your classes.
- **Form a community with teachers and other resource people.** You will find the support invaluable. Together with the researchers, the teachers participating in the NCISLA early algebra project committed to forming a professional development community. They met every month and focused their discussions on students' mathematical thinking, ways to elicit that thinking, how to figure out what students understood, and ways to engage students in discussing mathematical ideas. The teachers themselves were learners — learning about students' thinking and learning about mathematics.

For More Information

Elementary teacher resources and publications are available at www.wcer.wisc.edu/ncisla. See:

Teacher learning and professional development: The NCISLA fall 1998 newsletter *Principled Practice: Teachers as Learners*, at www.wcer.wisc.edu/ncisla.

Children's understanding of equality: *Children's Understanding of Equality: A Foundation for Algebra*, by Karen P. Falkner, Linda Levi and Thomas P. Carpenter, in *Teaching Children Mathematics*, vol. 6, no. 4, Dec. 1999. (Also at www.wcer.wisc.edu/ncisla/teachers)

Cognitively Guided Instruction: *Children's Mathematics — Cognitively Guided Instruction* (with two multimedia CDs), by Thomas P. Carpenter, Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. Heinemann Publishers, 1999.

ABOVE: An elementary student explains his reasoning to his teacher and fellow students.

ABOUT *in Brief*

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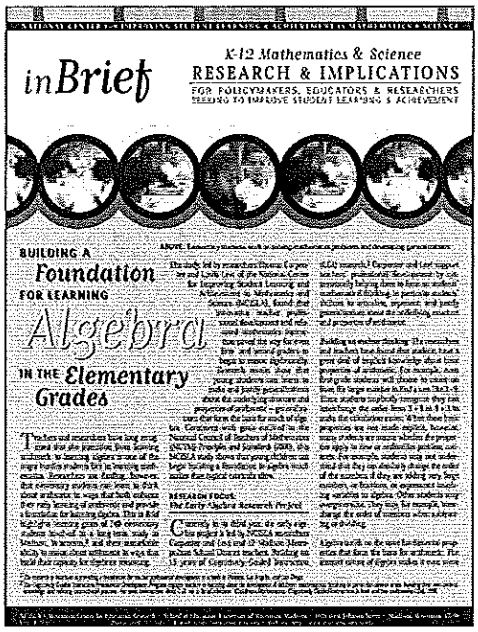
inBrief
K-12 Mathematics & Science
RESEARCH & IMPLICATIONS
FOR POLICYMAKERS, EDUCATORS & RESEARCHERS
BUILDING A
Foundation
FOR LEARNING
Algebra
IN THE Elementary
Grades

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NCISLA Director: Thomas P. Carpenter

NCISLA Associate Director: Richard Lehrer

NCISLA Communication Director: Susan Smetzer Anderson

Writers: Thomas P. Carpenter, Linda Levi, Valerie Farnsworth

Designer: Kristen Roderick

References

Carpenter, T. P., & Levi, L. (2000). Developing conceptions of algebraic reasoning in the primary grades (Res. Rep. 00-2). Madison, WI: National Center for Improving Student Learning and Achievement in Mathematics and Science. [Available at www.wcer.wisc.edu/ncisla]

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively Guided Instruction.* Portsmouth, NH: Heinemann.

Carpenter, T. P., & Franke, M. L. (1998, fall). Teachers as learners. *Principled Practice,* 2 (2). [Available at www.wcer.wisc.edu/ncisla]

Falkner, K. P., Levi, L., & Carpenter, T. P. (1999). Children's understanding of equality: A foundation for algebra. *Teaching Children Mathematics,* 6 (4), 232-6. [Available at www.wcer.wisc.edu/ncisla]

Kieran, Carolyn. (1981). Concepts associated with the equality symbol. *Educational Studies in Mathematics 12,* 317-26.

Matz, Marilyn. (1982). Towards a process model for school algebra errors. In D. Sleeman, & J. S. Brown (Eds.), *Intelligent tutoring systems* (pp. 25-50). New York: Academic Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

Saenz-Ludlow, A., & Walgamuth, C. (1998). Third graders' interpretations of equality and the equal symbol. *Educational Studies in Mathematics,* 35 (2), 153-87.

NATIONAL CENTER FOR IMPROVING STUDENT
LEARNING & ACHIEVEMENT IN MATHEMATICS & SCIENCE
Wisconsin Center for Education Research
School of Education, University of Wisconsin-Madison
1025 West Johnson Street, Madison, WI 53706