



Justification and Proof

Outcomes for Justification and Proof:

- Participants will consider how students articulate and refine justifications.

Big Ideas about Justification

Mathematicians have established quite rigorous standards of what counts as proof in mathematics. In elementary school, the term justification encompasses a broader range of arguments that children use to show that a conjecture is true. The arguments that students use to justify that their answers are correct are essentially justifications.

Justification is central to mathematics and even young children cannot learn mathematics with understanding, without engaging in justification. In order to make sense of the concepts and procedures they are learning, children need to justify the concepts and procedures to themselves.

Students' attempts to justify that mathematical statements are true can be separated into three broad classes:

- appeal to authority;
- justification by examples;
- generalizable arguments.

General forms of justification (generalizable arguments):

- restating the conjecture;
- concrete examples building on basic concepts;
- building on already justified conjectures.

Basic questions teacher can use repeatedly to encourage students to think about justifications:

- Is that always true?
- How do you know that it is true for all numbers?
- Okay, so we have seen that it works for a lot of numbers, but how do we know that there is not some number—maybe a very, very big number—that it will not work for?

A single counterexample is sufficient to demonstrate that a conjecture is not true for all numbers. Trying a variety of number is a good strategy for finding counterexamples.

Children are able to generate conjectures well before they are able to understand what constitutes appropriate justification. It is still worthwhile to engage students in conversations about how they know their conjectures are true. The interactions around these questions can yield some interesting insights about children's thinking.

Loose guidelines about when certain kinds of thinking will emerge from children:

- Most first and second graders rely on justification by example.
- By third or fourth grade, many children can begin to understand and use more general forms of argument.
- Fifth and sixth graders are able to engage in quite sophisticated discussion of justification.

The guidelines above are based on observations and it is important to remember that development will be based on previous experience. All students making justification for the first time are likely to use justification by example.