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Doing Algebra in Grades K–4

Zalman Usiskin

About thirty-five years ago the movement to incorporate geometry into the elementary grades began. To many elementary school teachers, the mention of the word *geometry* brought back memories of a high school geometry course that dealt with abstraction and proof. The thought of teaching children this geometry was naturally viewed with incredulity.

But in time teachers came to realize that the geometry being touted for the primary school deals with shape and size and drawing and counting. It fits in not only with the arithmetic that is the backbone of the curriculum but also with art and social studies. And much of its terminology, such as circle, triangle, square, and angle, are in our everyday language and part of basic literacy. Geometry today begins for many children in preschool.

Today, the word *algebra* inspires much the same reaction as *geometry* did in 1960. It spawns memories of "word problems," of complicated equations and expressions, of what may have seemed to be meaningless manipulations of meaningless symbols on a page. Even those teachers possessing great confidence in teaching arithmetic and geometric concepts and skills may shy away from mentioning any algebra. To many teachers, introducing algebra in the primary grades is the epitome of working with mathematical concepts too early, before students are ready.

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The thesis of this article is that it is neither unwise nor unproductive to do some algebra in grades K–4. The article introduces terms and shows that some algebra is being taught to, and learned by, virtually all students, even though the teacher may not realize it.

What Is Algebra?

Algebra is a language. This language has five major aspects: (1) unknowns, (2) formulas, (3) generalized patterns, (4) placeholders, and (5) relationships. Any time that any of these ideas are discussed, from kindergarten upward, there is opportunity to introduce the language of algebra.

Unknowns

Consider the following questions:

> What number, when added to 3, gives 7?
>
> Fill in the blank: 3 + ___ = 7
>
> Put a number in the square to make this sentence true: 3 + ❑ =7
>
> Find the ?: 3 + ? = 7
>
> Solve: $3 + x = 7$

In each question is an *unknown*. It is a number represented by that word, or by a blank, or a square, or a question mark, or the letter *x*. Some argue that only the last example is algebra. But only convention causes us to use x instead of ? or ❑ or ___ to represent an unknown. The verbal description of a situation, as in the first question, "What number, when added to 3, gives 7?" may seem to be the least algebraic, but it was the way that many people did algebra before the invention of modern symbolism in the 1590s. (The use of x and y to represent unknowns dates from Descartes in the early 1600s.) Thus, there is a sense that you are doing algebra whenever you ask students to find an unknown in a situation.

Formulas

If we have the formula $A = LW$ for the area of a rectangle and we ask students to find A when $L = 5$ and $W = 7$, we are doing algebra. If we ask students to find n when $5 \times 7 = n$, whether we are doing algebra is not clear.

If the teacher asks, "What number can I replace n by and make this a true statement?" the teacher is treating the statement as algebra. If the teacher asks, "What is the answer?" then the teacher is treating the question as arithmetic. The point is that much of the difference between arithmetic and algebra is in the ways questions are couched. It is not hard to do algebra, even with very young students.

Generalized Patterns

My father was a bookkeeper by trade, and he taught me a number of shortcuts for doing arithmetic. For instance, to multiply a number by 19, I could multiply the number by 20 and then subtract the number. The algebraic description is short. If n is the number, $19n = 20n - n$. This special case of the distributive property of multiplication over subtraction is called just the *distributive property* for short. Notice how much shorter the algebraic description is than the description in words. Furthermore, the algebraic description bears a visual resemblance to the arithmetic. For instance, if you buy 19 notebooks at \$2.95 each, substitute \$2.95 for n.

$$19 \cdot \$2.95 = 20 \cdot \$2.95 - \$2.95$$

Many people can calculate the right side using mental arithmetic. It equals \$59.00 – \$2.95, or \$56.05.

The algebraic description just given suggests that algebra is the most appropriate language for writing down general properties in arithmetic. You may tell students, "You can multiply two numbers in either order, and the answer will be the same," but you can write "For any numbers a and b, $a \cdot b = b \cdot a$." The specific instance $6 \cdot 12 = 12 \cdot 6$ looks like the algebra and does not look at all like the verbal description.

So you are doing algebra if you discuss generalizations such as "Add 0 to a number, and the answer is that number. Add a number to itself, and the result is the same as two times the number." But instead of writing them down in English, you use the language of algebra ($0 + n = n$; $t + t = 2t$).

Placeholders

Most people have played Monopoly or other board games in which the following kind of direction is given: "Roll the dice. Whatever number you get, move forward twice the number of spaces." In algebraic language it means "If you roll d on the dice, then move forward $2d$."

Spreadsheets use algebra. Take the number in one cell of an array, subtract it from a number in a second cell, and put the difference in a third cell. As in the dice situation, we do not need to know what numbers we have to understand the directions. If the number in the first cell is x and the number in the second cell is y, then the number in the third cell is $y - x$.

Consequently, whenever one plays a "pick a number" game—pick a number, add 3 to it, subtract 5, and so on—one is verbally doing algebra, for one is thinking of a number, any number, and dealing with it.

Relationships

Bob is two years older than Marisha. What could be their ages? If Marisha is 7, then Bob is 9. If Marisha is 4, then Bob is 6. We do not have to know their ages to know how they are related. If Bob's age is represented by B and Marisha's age is represented by M, then we could write the following:

$B = M + 2$ (Bob is 2 years older than Marisha.)

$B - M = 2$ (The difference in their ages is 2.)

$M = B - 2$ (Marisha is 2 years younger than Bob.)

Any of these representations is correct. Although there are many ways to write the relationship between B and M, they are equivalent. This equivalence is easier to determine in the algebraic descriptions than in the English descriptions in parentheses beside them.