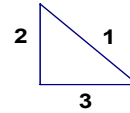


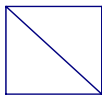
Shapes from Four Triangles Solution

Solve the problem by starting with a smaller problem. Determine how many possible shapes of two isosceles right triangles exist. Label the sides of the 1st triangle 1, 2, and 3.

The second triangle could be placed on one of three sides.

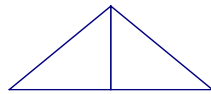


A square results if the second triangle is placed on side 1.

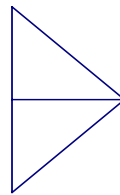


1

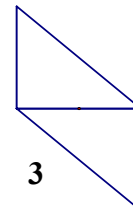
A “bigger” isosceles triangle results if the second triangle is placed on side 2 or 3. The second polygonal region is the first rotated 90° clockwise; hence only 1 new polygonal region is formed.



2



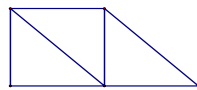
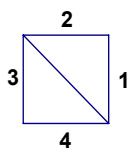
A parallelogram results if the triangle is rotated 180 about the midpoint of the bottom leg.



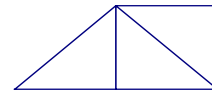
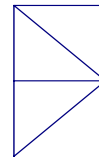
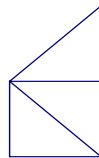
3

Therefore, there are 3 unique arrangements of 2 isosceles right triangles. Using the above arrangements, determine how many isosceles right triangle arrangements with 3 triangles are possible.

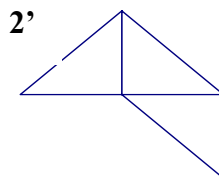
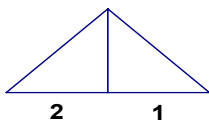
With the square arrangement, the third triangle could be placed on 4 different sides, but only 1 unique polygonal shape results. Rigid motions (rotation, reflection, glide reflection) determine the last three (not numbered) polygonal regions are not different shapes.



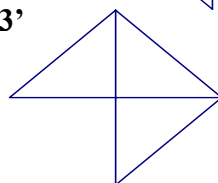
1'



With the “bigger” isosceles triangle, the third triangle can be positioned to form two different arrangements.



2'

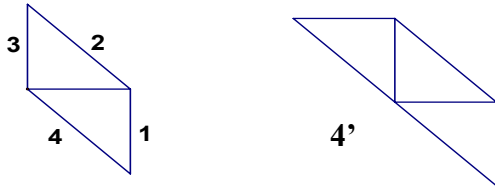


The third triangle is placed on side 1 to form the first polygonal region with the “bigger” isosceles triangle. When the triangle is flipped, a second unique polygonal region is formed. If side 2 were used, rigid motions would show the regions are not different from the two already formed.

With the parallelogram, the third triangle will form one unique arrangement.

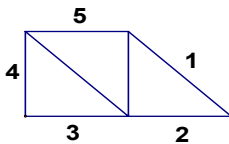
If the triangle is placed on side 1 or 3, an isosceles trapezoid shape is formed.

If the third triangle is placed on sides 2 or 4, the resulting polygonal shape is congruent to the square arrangement.

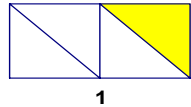


Therefore, there are 4 unique arrangements of 3 isosceles right triangles.

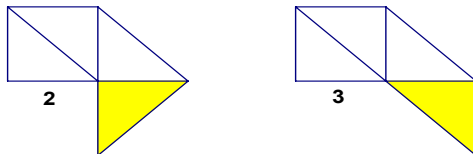
Now the number of unique arrangements of 4 isosceles right triangles can be determined. Begin with the square arrangement of three isosceles right triangles. There are 5 sides where the 4th triangle can be placed.



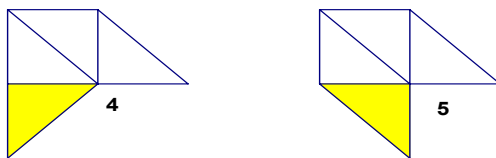
If the 4th triangle is positioned on side 1, the result is a rectangle.



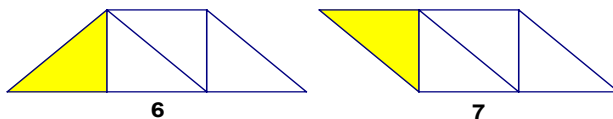
The 4th triangle can be positioned 2 different ways on side 2.



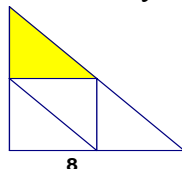
The 4th triangle can be positioned 2 different ways on side 3.



The 4th triangle can be positioned 2 different ways on side 4.

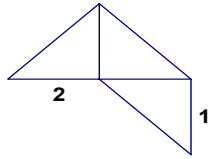


The 4th triangle can be positioned 1 way on side 5.

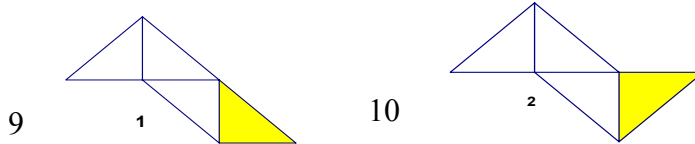


Therefore, there are 8 different arrangements that use the square shape.

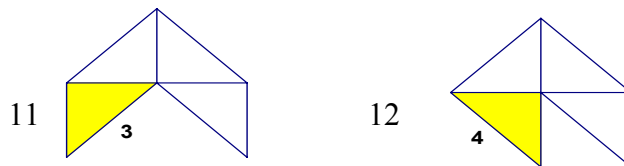
With the first “bigger isosceles” triangle arrangement, 2’, the 4th triangle can be positioned on 2 different sides to form unique shapes.



The 4th triangle can be positioned 2 different ways on side 1.

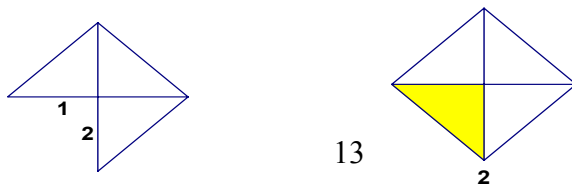


The 4th triangle can be positioned 2 different ways on side 2.



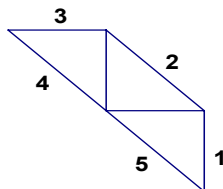
There are 4 different ways the 4th triangle can be positioned using the 1st “bigger” isosceles triangle arrangement.

With the 2nd “bigger” isosceles triangle shape, 3’, only one position of the 4th triangle will result in a unique shape. This occurs when the legs of the 4th isosceles right triangle are placed on sides 1 and 2.

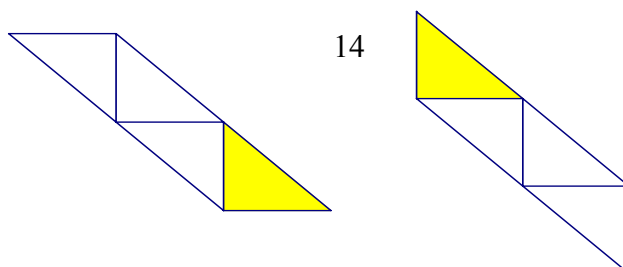


Therefore, using the “bigger” isosceles triangle arrangements, 5 new polygonal shapes are formed.

With the parallelogram arrangement, the 4th triangle will form one *unique* shape.



If the triangle is placed on side 1 or 3, the last new arrangement is formed. Again, rigid motions (a rotation) show that the two polygonal arrangements are congruent.



Hence, the total number of unique arrangements of 4 isosceles right triangles is 14!