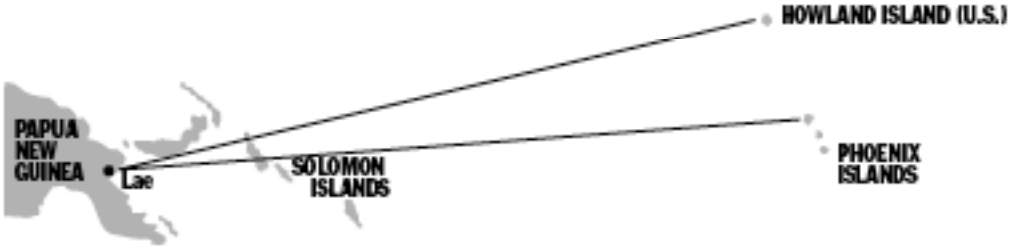
The Amelia Earhart Problem

Situation 1: Amelia Earhart, a pioneer in aviation, was lost in a plane crash because of a navigation error when she was flying around the world. Her navigator made a 7.5 degree error. The illustration below shows both her real destination and where she actually is thought to have crashed.



Adapted from Small Blue Planet. © Copyright 1993–1995 by Now What? Software, San Francisco. The original map was created from maps made by the Central Intelligence Agency and the Defense Mapping Agency.

The teacher asked, "Was she off very much as far as the angle is concerned?" and most of the class said, "No." But Claire argued, "But that's like on a small scale though. In real life it would be really big." John agreed and added, "It would be like 200 degrees." The teacher asked, "What do you mean by that?" and Claire answered, "That's a small map. If you made the map bigger, if you blew it up bigger, it would be a different distance, because [the islands] wouldn't be so scrunched together. It would be farther apart. And if it was real life, it would be a really big angle."

Do you agree or disagree with Claire?

Keiser, J.M. et al. (2003). An Assessment of Students' Understanding of Angle, *NCTM/MTMS Journal, 9(2), 116-119*

Research Summary
Challenging Idea: Angle

Angle Definition
When defining angle, most students' definitions do not incorporate the dynamic notion of rotation. Instead angles are defined by their measure or by referring to the elements involved in an angle like line segments, rays, an intersection point, and/or the area between the rays.

Angle Measure
When students measure angles, once again, many do not think in terms of rotation. They search for a way to apply their previous knowledge of static measures (points, length, area) to situations of angle measure. Some students who attempt to apply static measures to angle decide the linear distance between the two rays/segments of the angle determines angle measure. Therefore, to them, angle size depends on the length of the rays/segments and which points along the rays/segments are used to perform the measurement.

Evidence that angle is a challenging idea for students:
Keiser, Klee, and Fitch (2003) asked 77 sixth-grade students to define angle in their own words. Definitions generated by students were classified according to their emphasis and the table below shows that no students made reference to the idea of rotation or turn, and that most either defined angle in terms of its measure or its static components.

Emphasis of Definition	Percent of students
The degrees or the measure itself "An angle is the number of degrees in a corner"	29.5%
The line segments that meet "An angle is where two line segments intersect to form an angle"	25.6%
The opening "How big apart the two lines are apart where the vertex is"	7.7%
The point "An angle is where two vertices meet and make a point"	7.7%
Measure of the edge "An angle is the measure of one side of a shape"	5.1%
Combination of two of these "I think an angle is the measure in degrees between two line segments that are touching"	10.3%
Vague or wrong statements "An angle is a shape that has straight lines and at least 3 angles"	14.1%

The Amelia Earhart Problem—the results:
Seventy-six percent of students agreed with Claire, suggesting that students struggled to understand that when an angle is "scaled up", or enlarged, the measure stays the same. In response to this question, one student wrote, "Yes (I agree), the map is a small version even though it may look like an actual picture. If you blew it up, it would be a bigger angle." Another students responded, "I agree because a bigger picture is a bigger degree." This, along with the definitions students provided earlier, suggests that these students are thinking of angle measure as a linear distance or area between rays, and not a measure of rotation to move from one ray to the other.

Students' difficulties in defining and measuring angles
Definitions students encounter often focus on one of the following ideas—the union of two rays with a common endpoint (static), the region contained between the two rays (static), or the turning of a ray about a point from one position to another (dynamic)—with the first two much more common than the third. Static representation call for linear, area or volume measures of some sort, and so students respond accordingly.

Students may incorrectly think that the linear distance between two rays determines the angle measurement; therefore, the measure would change depending on the points on the rays used to determine this distance.

Or, they incorrectly think that the longer the rays the larger the angle. This leads to difficulty in understanding the effects scaling has on angle (as in *The Amelia Earhart Problem*).

Considering the area between the rays as determining the angle measure is another common misconception. Since the area between the rays again depends on the length of the rays, these students are also led astray.

Adapted from *Fostering Geometric Thinking*
Education Development Center