From NCTM's Archives

Computational understanding and proficiency are in many ways the quintessential goals of elementary school mathematics. Although the terminology is somewhat new, the idea of computational fluency has deep historical roots. We searched the archives of *Teaching Children Mathematics* and its predecessor, *The Arithmetic Teacher*, for an article that would give readers a sense of the timelessness of some of the issues surrounding the work of teaching for computational fluency. The following piece by William Brownell, published very early in the history of the *AT*, provides a wonderful balance between historical perspective and contemporary relevance.

In his article, Brownell explains why progress toward "meaningful habituation"—in many ways a historical precursor to computational fluency—was stifled and advocates an instructional approach that balances meaning and skill. Brownell makes explicit the fact that meaning and skill are mutually dependent, even though some people attempt to portray them as distinct. As do current focus issue articles, this nearly fifty-year-old piece argues in favor of computational sense making and exploration. It stresses the importance of foundational elements such as place value and the need for students to increase computational efficiency over time. Brownell and several other authors in this focus issue also allude to the essential role of student and teacher discourse in the development of computational fluency.

Perhaps most important, this article prompts contemporary readers to consider relevant aspects of teaching for computational fluency that other focus issue articles do not address. Considering that we inhabit an educational setting dominated by calls for accountability, it is somewhat surprising that Brownell is the only author in this focus issue to note the instructional dilemmas that arise when standardized testing and computational competence meet. His view that classroom-based assessment is at the core of teaching for computational competence is an important message for today's educators. Just as useful is Brownell's pairing of the need to support teacher learning with a professional willingness to fully explore and implement practices that are necessary to foster meaningful computational proficiency.

The field of mathematics education has made a great deal of progress since Brownell's time. Constructivist and sociocultural theories have replaced conditioning and field theory in today's literature. A greater variety of materials is available to support teacher learning, and the alternative assessment movement has made many more tools available for classroom use. In 1956, Brownell identified the following hurdles to fostering meaningful habituation:

- A failure to emphasize the role of the "right" kind of practice in learning
- The hazy implications of learning theory for teaching
- Incomplete teacher understanding of practices that support meaningful proficiency

His analysis of these hurdles can serve as a roadmap for those who work today to ensure that computational fluency becomes a part of all children's futures.—*Timothy A. Boerst and Jane F. Schielack, focus issue editors* ▲

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Meaning and Skill—Maintaining the Balance*

WILLIAM A. BROWNELL
University of California

THE SUBJECT GIVEN ME poses a question. It is no academic question arising out of purely theoretical considerations. It is assumed that both meaning and computational competence are proper ends of instruction in arithmetic. It is implied that somehow or other both ends are not always achieved and that there is evidence that this is so.

Indeed there is such evidence. More than one school system has embarked upon a program of so-called meaningful arithmetic, only to discover that on standardized tests of computation and "problem solving" pupils do none too well. In such schools officials and teachers are likely to believe that they have made a bad bargain; and school patrons are likely to support them vehemently in this belief.

We may try to convince all concerned that the instruments used to evaluate learning are inappropriate, or at least imperfect and incomplete. True, standardized tests rarely if ever provide means to assess understanding of arithmetical ideas and procedures. Hence, the program of meaningful instruction, even if well managed, has no chance to reveal directly and explicitly its contribution to this aspect of learning. On the other hand, can we deny that the learning outcomes that *are* measured are of no significance? To do so is to say in effect that computational skill is of negligible importance, and we can hardly justify this position.

Why is there now the necessity to talk about establishing and maintaining the desirable kind of balance between meaning on the one hand and computational competence on the other?

Sources of the Dilemma

1. Incomplete Exposition of "Meaningful Arithmetic"

Perhaps those of us who have advocated meaningful learning in arithmetic are at fault. In objecting to the drill conception of the subject prevalent not so long ago, we may have failed to point out that practice for proficiency in skills has its place, too. It is questionable whether any who have spoken for meaningful instruction ever proposed that children be allowed to leave our schools unable to compute accurately, quickly, and confidently. I am sure that all, if asked, would have rejected this notion completely. But we may not have *said* so, or said so often enough or vigorously enough. Our comparative silence on this score may easily have been misconstrued to imply indifference about proficiency in computation.

If this has actually happened, we can scarcely blame classroom teachers if they have neglected computational skill as a learning outcome. It is characteristic of educational movements to behave pendulum-wise. When we correct, we tend to overcorrect. Just this sort of thing seems to have happened in the teaching of arithmetic. In fleeing from over-reliance on one kind of practice, we may have fled too far. It is a curious state of affairs that those of us who deplored the limitations of this kind of practice must now speak out in its behalf and stress its positive usefulness.

2. Misunderstood Learning Theory

A second possible explanation for our dilemma may be found, as is frequently the case in the practical business of education, in misinterpretations or misapplications of psychological theories of learning. Over-simplification of certain generalizations in learning theories as widely apart as are those of conditioning and of field theory could lead, and may have led, to the lessening of emphasis on practice in arithmetic.

a. Conditioning theory. According to the learning theory of one influential exponent of conditioning, once a response has been made to a stimulus a

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connection has been established. Thus, the child who says "Seven" in replying to the question, "How many are two and five?", sets up a connection between 2 and 5 as stimulus and 7 as response. If this is so—if the connection is actually formed by the one response—then at first glance practice might seem to be utterly purposeless.

But this inference is quite unfounded, as the psychologist in question makes abundantly clear. In saying that a connection is "established" he means no more than that a new neuromuscular pattern is available. He does *not* mean that our hypothetical child after the one experience will always, only, and instantly respond with "seven" when asked the sum of 2 and 5. If the response "seven" is to be the invariable one, moreover if the response is to be made in situations differing ever so slightly from the original one, then practice is required. In this theory of learning as conditioning, therefore, there is no comfort for those who would abandon practice as a means to promote the learning of arithmetical facts and skills.

b. Field theory. No more comfort is to be found in field theory of learning. It is often said that one experience of "insight" or "hindsight"—before, during, or after success—is enough; but enough for *what*? It may be all that is needed to understand a situation and the method of dealing with it. Yet, it is one thing to know the general rationale for solving a complicated mechanical puzzle but quite another thing to be able to manipulate the parts correctly with facility, ease, and speed.

So in arithmetic it is one thing to comprehend the mathematical principles governing decomposition in subtraction—something that can come from a single insightful experience—but another thing to be able to subtract quickly and correctly. Indeed if in examples like 73 – 47 and 52 – 19 a child who possesses this understanding always thinks through the complete logical explanation, his performance will be impaired, at least from the standpoint of speed. Understanding and skill are not identical. A single instance of insight may lead to understanding but will hardly produce skill. For skill, practice is necessary.

3. Influence of General Educational Theory

A third explanation for failure to stress computational competence is to be found in the recent history of educational theory. It must be remembered that the place of meaning and understanding in arithmetic has been generally recognized for not more than fifteen or twenty years. In 1935 we were still under the influence of somewhat sentimental and unrealistic notions both about children and about the course of their development. In extreme form these notions led to a kind of teaching that was anything but systematic. Indeed, what children learned, when they learned it, and how they learned it was left pretty much to the children themselves. Attempts to guide and direct learning and to organize learning experiences were frowned upon as "violating child nature" and as almost certainly productive of serious derangements of child personality. To those who held these views practice was anathema.

In the public schools, as contrasted with college departments of Education, this conception of the processes of learning and teaching did not gain much of a foothold. Nevertheless, it was in this climate of thought that stress on meaning in arithmetic put in its appearance. Those who were committed to the educational theory I have mentioned welcomed the new emphasis as confirming both what they did and what they did not do. If, deliberately or unwittingly, they accepted only the part of the emerging view most congenial to them, they committed an error that is very common and that is altogether human. Be that as it may, the consequences were none too good. True, these teachers may have been largely responsible for the quick and general endorsement of one aspect of meaningful arithmetic—learning with understanding. On the other hand, they could not themselves absorb the whole of it, and they remained hostile to practice in learning. Pupils taught by such teachers cannot be expected to make high scores on standardized arithmetic tests of skill in computation and "problem solving."

4. Inadequate Instruction on Meanings

I have suggested three hypotheses as explaining why we may not be obtaining balance between understanding and computational competence in arithmetic, on the assumption that our shortcomings relate to the latter (computational competence) rather than to the former. These three are: the possible failure of advocates of meaningful arithmetic to emphasize sufficiently the importance of practice in acquiring arithmetical skills; misinterpretations of psychological theories of learning which have had the effect of minimizing the place of practice; and the unwillingness of some teachers, who believe completely that arithmetic must be made intelligible to children, to provide the practice necessary for computational proficiency. May I add a fourth? It is that we may not as yet be doing a very good job in teaching arithmetical meanings as they should be taught.

There is ample evidence in psychological research on learning that the effects of understanding are cumulative. There is also ample evidence, if not in arithmetic, then in other types of learning, that the greater the degree of understanding, the less the amount of practice necessary to promote and to fix learning. If these truths are sound—and I think they are—then they should hold in the field of arithmetical learning. It follows that computational skills among school children would be greater than they are if we *really* taught them to understand what they learn.

Again I remind you that meaningful arithmetic, as this phrase is commonly used, is a newcomer in educational thought. Many teachers, trained in instructional procedures suitable, say, to a view of arithmetic as a tool or a drill subject, find it difficult to comprehend fully what meaningful arithmetic is and what it implies for the direction of learning. Others than myself, I am sure, have, in conferences with teachers, been somewhat surprised to note that some of them are unfamiliar with major ideas in this conception and with methods of instruction adapted thereto.

Perhaps the commonest instructional error is, in a different context, the same one that has always distorted learning in arithmetic, namely, the acceptance of memorized responses in place of insistence upon understanding. Mathematical relationships, principles, and generalizations are couched in language. For example, the relationship between a given set of addends and their sum is expressed verbally in some such way as: "The order of the numbers to be added does not change the sum." It is about as easy for a child to master this statement by rote memorization as to master the number fact, 8 – 7 = 1, and the temptation is to be satisfied when children can repeat the words of the generalization *verbatim*. Similarly, the rationale of computation in examples such as: 33 + 48 and 71 – 16, makes use of concepts deriving from our number system and our notions of place value. But many a child glibly uses the language of "tens" and "ones" with no real comprehension of what he is saying. Such learning is a waste of time. To use an Irish bull, the meanings have no meaning.

I intend no criticism of teachers. Until recently there have been few professional books of high quality to set forth the mathematics of arithmetic and to describe the kind of instruction needed. Moreover, many teachers have had no access to these few books. Again, until recently not many courses of study and teachers' manuals for textbook series have been of much help. It is not strange, therefore, that though meaningful arithmetic is adopted in a given school system, not all members of the teaching staff are well equipped to teach it. As a result, their pupils, denied a full and intelligent treatment of arithmetic as a body of rational ideas and procedures, have been unable to bring to computation all the aid that could come through understanding.

Toward a Solution

So much for possible explanations—explanations of a general character—for our failure to keep meaning and skill in balance. I have no way of knowing the reality of any of the four hypotheses suggested or of the extent of its validity, to say nothing of the degree to which, taken together, the four account adequately for the situation. The fact remains that something needs to be done. What is the remedy?

Certainly we shall not get very far as long as we think of understanding and practice in absolute terms. I have deliberately done this so far in order to examine the issue in simple terms. Actually, it is erroneous to conceive of understanding as if it were either totally present or totally absent. Instead, there are degrees or levels of understanding. Likewise, not all forms of practice are alike. Rather, there are different types, and they have varying effects in learning.

Levels of Understanding

Consider the example 26 + 7. The child who first lays out twenty-six separate objects, next seven more objects, and then determines the total by counting the objects one by one has a meaning for the operation. So has the child who counts silently, starting with 26. So has the child who breaks the computation into two steps, 26 + 4 = 30 and 30 + 3 = 33. So has the child who employs the principle of adding by endings,—6 + 7 = 13, so 26 + 7 = 33. So has the child who, capable of all these types of procedure, nevertheless recognizes 33 at once as the sum of 26 and 7. All these children "understand," but their understandings may be said to represent different points in the learning curve. The counter is at the bottom, and the child who through understanding has habituated his response "thirty-three" so that it comes automatically is at the top of a series of progressively higher levels of performance.

All these levels of performance or of understanding are good, depending upon the stage of learning when they are used. For instance, finding the sum of 26 and 7 by counting objects is a perfectly proper way of meeting the demand at first; but it is not the kind of performance we want of a child in grade 4. At some time in that grade, or earlier, he should arrive at the stage when he can announce the sum correctly, quickly, and confidently, with a maximum of understanding.

It is a mistake to believe that this last stage can be achieved at once, by command as it were. When a child is required to perform at a level higher than he has achieved, he can do only one of three things. (a) He can refuse to learn, and his refusal may take the form; "I won't," or "I can't," or "I don't care," the last named signifying frustration and indifference which we should seek to prevent at all costs. (b) Or, he can acquire such proficiency at the level he *has* attained that he will be credited for thinking at the level desired. Many children develop such expertness in silent counting that, in the absence of close observation and questioning, they are believed to have procedures much beyond those they do have. (c) Or, third, he may try to do what the teacher seems to want. If an immediate answer is apparently expected, he will supply one, by guessing or by recalling a memorized answer devoid of meaning. If he guesses, obviously he makes no progress at all in learning. If he memorizes, only unremitting practice will keep the association alive; and if he forgets, he is helpless or must drop back to a very immature level of performance such as counting objects or marks by 1's.

For the stage of performance we should aim for ultimately, as in the case of the simple number facts, higher-decade facts, and computational skills, we have no standard term. We may use the word "memorization" to refer to what a child does when he learns to say "Four and two are six" without understanding much about the numbers involved, about the process of addition, or about the idea of equivalence. If we employ "memorization" in this sense, then that word is inappropriate for the last step in the kind of learning we should foster. Hence for myself I have adopted the phrase "meaningful habituation." "Habituation" describes the almost automatic way in which the required response is invariably made; "meaningful" implies that the seemingly simple behavior has a firm basis in understanding. The particular word or phrase for this last step in meaningful learning is unimportant; but the *idea*, and its difference from "memorization," *are* important.

Teaching meaningfully consists in directing learning in such a way that children ascend, as it were, a stairway of levels of thinking arithmetically to the level of meaningful habituation in those aspects of arithmetic which should be thoroughly mastered, among them the basic computational skills. Too many pupils, even some supposedly taught through understanding, do not reach this last stage. Instead, they stop short thereof; and even if they are intelligent about what they do when they compute, they acquire little real proficiency. In instances of this kind both learning and teaching have been incomplete.

How are teachers to know the status of their pupils with respect to progress toward meaningful habituation? Little accurate information is to be had from their written work, for both correct and incorrect answers can be obtained in many ways, and inference is dangerous. Insightful observation and pupils' oral reports volunteered or elicited through questioning are more fruitful sources of authentic data. Since children differ so much in their thought procedures, a good deal of this probing must be done individually. One of the most fruitful devices I can suggest for this probing consists in noting what children do in the presence of error.

I recall a conversation with a fourth grade girl whom I knew very well and who was having difficulty in learning—in her case, in memorizing—the multiplication facts. I asked her—her name was June—"How many are five times nine?" (This form of expression was used in her school instead of the better "How many are five nines?") Immediately she responded, "Forty-five." When I shook my head and said, "No, forty-six," she was clearly upset. Her reply, after some hesitation, was, "No, it's forty-five." When I insisted that the correct product is 46, June said, "Well, that isn't the way I learned it." I suggested that perhaps she had learned the wrong answer. Her next statement was, "Well, that's what my teacher told me." This time I told her that she may have misunderstood her teacher or that her teacher was wrong. June was obviously puzzled; then she resorted to whispering the table, "One times nine is nine; two times nine is eighteen," and so on, until she reached "five times nine is forty-five." Again I shook my head and said, "Forty-six." When she was unable to reconcile my product with what she had become accustomed to say, I asked her, "June, have you no way of finding out whether forty-five or forty-six is the correct answer?' Her response was, "No, I just learned it as forty-five." Of course I did not leave her in her state of confusion; but the point of the illustration is, I hope, quite apparent: Her inability to deal with error was convincing evidence of the superficiality of her "learning" and of its worthlessness.

Compare my conversation with June with that I had with Anne, another fourth grade girl whom also I knew well and who, like the first girl, was learning the multiplication facts. When I asked Anne, "How many are five times nine?", the correct answer came at once, just as in June's case; but from here on, mark the difference. I introduced the error, saying that the product is 46, not 45. Anne looked at me in disgust and said, "Are you kidding?" I maintained my position that $5 \times 9 = 46$. Immediately she said, "Do you want me to prove it's forty-five?" I told her to go ahead if she thought she could. She answered, "Well, I can. Go to the blackboard." There I was instructed to write a column of five 9's and, not taking any chances with me, Anne told me to *count* the 9's to make sure I had 5. Next came the command, "Add them."

When I deliberately made mistakes in addition, she corrected me, each time saying, “Do you want me to prove that, too?” Obviously, I had to arrive at a total of 45. Having done so, I said, “Oh, that’s just a trick,” to which she replied, “Do you want me to prove it another way?” Exposure to error held no terrors for this child; she did not become confused or fall back upon repetition of the multiplication table; nor did she cite her teacher as an authority. Instead, Anne had useful resources in the form of understandings that were quite lacking in the case of June whose discomfiture I have described.

Probing for understanding need not depend wholly on opportunities to work with individual children. On the contrary, there are possibilities also under conditions of group instruction when questions beginning with “How” and “Why” supplement the commoner questions starting with “What.” The worth of valid knowledge concerning level of understanding is inestimable for the guidance of learning. The demands upon time are not inconsiderable, but no one should expect to get full knowledge concerning every pupil. The prospects are not hopeless if ingenuity is exercised and if the goal is set, not at 100% of knowledge, but more realistically at perhaps 20% more than is now ordinarily obtained.

Types of Practice

In crude terms, practice consists in doing the same thing over and over again. Actually, an individual never does the same thing twice, nor does he face the same situation twice, for the first reaction to a given situation alters both the organism and the situation. A changed being responds the second and the third time, and the situation is modified accordingly.

We must concede the truth of these facts. At the same time, for our purposes we may violate them a bit. Let us conceive of practice of whatever kind as falling somewhere along a continuum. At the one end of the continuum is practice in which the learner tries as best he can to repeat just what he has been doing. At the other end is practice in which the learner modifies his attack in dealing with what is objectively the same situation (or what to him are similar situations). We may call these extremes “repetitive practice” and “varied practice,” respectively.

An instance of repetitive practice is memorizing the serial order of number names by rote or applying the series in the enumeration of groups of objects. An instance of varied practice is the attempt, by trying different approaches, to find steadily better ways of computing in such examples as 43 + 39, 75 – 38, 136 ÷ 4, and 32 × 48. Between the two extreme types of practice are innumerable others, differing by degree in the extent to which either repetition or variation is employed. But again, for our purposes, we may disregard all the intervening sorts of practice: we could not possibly name them all, or describe them, or show their special contributions of learning.

Both repetitive and varied practice affect learning, but in quite unlike ways. For illustration we may choose an instance of learning outside of arithmetic, for example, the motor activity of swimming. Suppose the beginner engages in repetitive practice: What does he do, and what will happen? Well, he will continue to use as nearly as he can exactly the movements he employed the first time he was in deep water, and the result will be that he may become highly proficient in making just those movements. He will hardly become an expert swimmer, but he will become an expert in doing what he does, whether it be swimming or not.

On the other hand, suppose that the beginner engages in varied practice. In this case he will seek to *avoid* doing precisely what he did at the outset. He will discard uneconomical movements; he will try out other movements, select those that are most promising, and seek a final coordination that makes him a good swimmer. Then what will he do? He will change to repetitive practice, for, having the effective combination of movements he wants, he will seek to perfect it in order to become more proficient in it.

The differences between repetitive practice and varied practice, both in what the learner does and in what his practice produces, are clearly discernible when we think of motor activities. They are less easily identified when we think of ideational learning tasks like the number facts and computational skills. But the differences are there none the less. The child who counts and only counts in dealing with examples like 36 + 37 and 24 + 69 is employing repetitive practice. The more he counts, the more expert he becomes in counting; but the counting will not, and cannot, move him to a higher level of understanding and of performance. In contrast is the child who, through self-discovery or through instruction, tries different ways to add in such examples. Under guidance he can be led to adopt higher and higher levels of procedures until he is ready for meaningful habituation. If then he does not himself fix his automatic method of adding, he can be led to do so through repetitive practice. In any case it is safer to provide the repetitive practice in order to increase proficiency and make it permanent.

The distinction between repetitive and varied practice, in their nature and in their consequence, is not always recognized in teaching. If repetitive practice is introduced too soon, before understanding has been achieved, the result, for one thing, may be blind effort and frustration on the part of the learner. Or, it may fix his performance at a low level, the level he has attained. No new and better procedure can emerge from repetitive practice though it may appear under conditions of drill when a child, tired of repetition or disappointed in its result, abandons it in search of something new.

There may be an instructional error of another kind, one already alluded to. This error is to insist, to quote some, that "there is no place for drill in the modern conception of teaching." True, there is no place for unmotivated drill on ill-understood skills; but the statement goes too far in saying that there is no place at all for repetitive practice. How else, one may ask, is the final step of meaningful habituation to be made permanent; how else is real proficiency at this level of learning to be assured?

The kind of practice most beneficial at any time, then, is the kind best adapted to accomplish a given end. For illustration, let us return to June and Anne and their learning of the multiplication facts.

June was trying to master these facts by repetitive practice, by saying over and over and over again the special grouping of words for each separate fact. Her level of understanding of the numbers and relationships involved was close to zero. Unless engaged in almost ceaselessly, her repetition of verbalizations could give her little more than temporary control, and control, be it noted, of the verbalizations alone. Lapses of memory would be nearly fatal and would subject her to the hazards of guessing. In no way could her memorization of senseless phrases contribute much to sound learning of the facts themselves, not to mention the deficiencies of its results for more advanced forms of computation and functional use. What June needed was not repetitive, but varied practice. By contrast, Anne, who was able to "prove" her announced products and who thereby demonstrated her full understanding of the relationship of the numbers, no longer needed varied practice and could safely and properly be encouraged to engage in repetitive practice.

We employ varied practice, then, if we wish the child to move upward from where he is toward meaningful habituation, and repetitive practice if we are endeavoring to produce true competence, economy, and permanence in this last stage of learning (or at any earlier stage which represents a type of performance of worth in itself). Practice has to be designed to fit the learner's needs, a fact which brings us back again to the individual and to the critical importance of accurate knowledge concerning his learning status.

In Conclusion

To sum up, the balance between meaning and skill has been upset, if indeed it ever was properly established. The reasons are many, some of them relating to educational theory in general, others to misconceptions of psychological theories of learning, others to failure to teach arithmetical meanings thoroughly, and still others to carry learning in the case of computation to the level I have denoted meaningful habituation, and then to fix learning at that level. I have discussed these matters at length, perhaps at unwarranted extent in view of the fact that the remedy for the situation can be stated briefly. The remedy I propose is as follows:

1. Accord to competence in computation its rightful place among the outcomes to be achieved through arithmetic;
2. Continue to teach essential arithmetical meanings, but make sure that these meanings are just that and that they contribute as they should to greater computational skill;
3. Base instruction on as complete data as are reasonably possible concerning the status of children as they progress toward meaningful habituation;
4. Hold repetitive practice to a minimum until this ultimate stage has been achieved; then provide it in sufficient amount to assure real mastery of skills, real competence in computing accurately, quickly, and confidently.

EDITOR'S NOTE [1956]. Dr. Brownell is talking particularly to teachers in school systems that have initiated programs of meaningful arithmetic but his message is for all teachers. He indicates that it is difficult to comprehend fully what meaningful arithmetic is. He warns against teaching for memorized generalizations under the guise of teaching meanings. A good sound program of arithmetic will develop meanings and understandings as basic and necessary to a functional program which does not slight computation and problem solving. In this country we study arithmetic primarily for its contribution to intelligent and successful living in social, economic, and cultural situations. We do not thereby neglect those mathematical elements which give substance to arithmetic as a science, We are not primarily concerned with arithmetic as a puzzle for the amusement of the few. In our society the abilities to read and to form mathematical conclusions are very important.