STRATEGIES FOR

Basic-Facts Instruction

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In a class of beginning second graders, children explained with these typical replies how they solved the number fact 8 + 7: "I know 7 + 7 is 14, and 1 more is 15"; "8 + 2 makes 10. But 7 has 5 more, so the answer is 15"; and "I just knew the answer was 15." Teaching basic number facts like 8 + 7 has been a goal of elementary mathematics instruction for more than 100 years and continues to be important today.

Although most teachers agree that students' fact mastery is important, many are unclear about how to seek it in ways that are consistent with the NCTM's Standards (1989, 1991, 1995). They even disagree about what knowing the basic facts means and when, or even if, students should achieve mastery. Is it appropriate to expect first graders to memorize addition facts, or will this task interfere with their mathematical thinking? What classroom practices can build both understanding and quick recall? Can fact mastery be achieved through problem-solving activities, or is practice necessary? If current reforms in mathematics education are to succeed, questions about the basic facts need answers.

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Why Should Children Learn the Facts?

Most people recognize that children should learn the basic facts because knowing them is useful, both in school and in life out of school. Estimation and mental computation require the use of basic facts. How can students use 80×40 to estimate 84×41 if they do not know 8×4? Students who know their facts do better in school mathematics. Parents, teachers, and the public expect schools to teach the basic facts.

Fortunately, no conflict need exist between fact mastery and school mathematics reform. Many goals of reform—helping students make connections between school mathematics and the real world, helping students develop conceptual understanding as well as procedural skills, helping students learn to explain their thinking and to understand others' explanations—can be achieved through a program that also leads to fact mastery. Properly approached, the basic facts offer excellent opportunities for teaching children to think mathematically.

How Should the Facts Be Taught?

The traditional rote approach to the basic facts, with frequent drill and timed tests, has serious disadvantages. Premature demands for quick performance can induce anxiety and undermine understanding. Rigid schedules for mastery do not accommodate individual differences and have unfortunate outcomes for some children. The rote approach encourages students to believe that mathematics is more memorizing than thinking.

Today, however, the outlines of a better approach are clear. This approach begins with children's natural thinking. The essence of many current reforms in primary-grade mathematics, including the approach to the basic facts described here, is to recognize and build on the wealth of informal mathematical knowledge that children bring to school. Traditionally, much of this knowledge has been ignored or suppressed.

Early work with basic facts should help children refine and extend their natural strategies for solving simple problems. As children increase their proficiency at various strategies, they begin to remember the simplest facts. Knowing the simpler facts makes possible more efficient strategies for harder facts. Gradually, students master more and more efficient strategies and commit more and more facts to memory. At the end of the process, students can accurately and automatically produce all the basic number combinations. Many of these combinations are recalled from memory, but a few may also be found through quickly executed strategies or suitable rules. Fact strategies and recall are used by both children and adults. We know a research mathematician, for example, who solves 8×7 by doubling 7 three times—14, 28, 56—but he does the doubling so quickly and effortlessly that it is automatic.

In this section of the article, we describe this "strategies" approach for the addition and subtraction facts; many of the same ideas also apply to the multiplication and division facts. First we describe how children's informal knowledge, especially their knowledge of counting and of part-whole relationships, can be used in beginning fact work. Then we describe how children use facts that they know to derive facts that they do not know. Finally, we discuss the role of practice and sketch a possible sequence for addition- and subtraction-facts instruction.

No conflict need exist between fact mastery and mathematics reform

Counting to solve problems

Perhaps the best way to extend primary-grade children's informal understanding of addition and subtraction is by asking them to solve simple problems—without telling them how those problems are to be solved. These problems can come from real life, classroom situations, textbooks, the teacher, or the children's imaginations: "An adult movie ticket is $7 and a child's ticket is $4. How much for one adult and one child?" "Miriam wants a toy horse that costs $15. She has $8. How much more does she need?"

As students encounter the problems, they should be encouraged to devise their own solution procedures by looking for patterns, thinking logically, and using manipulatives. The adult approach—reducing such problems to addition or subtraction number sentences and retrieving the answers from memory—is not a natural strategy for young children. Instead, primary-grade children tend to use direct modeling, counting, and derived-fact strategies (Bergeron and Herscovics 1990).

Direct-modeling techniques are generally the first to appear for a given type of problem. The child counts out objects to represent the quantities in a problem, performs actions with the objects that parallel the problem situation, and counts some set to find the answer. To solve the preceding movie-ticket problem, for example, a child might count out seven chips for the cost of the adult's ticket and four chips for the child's ticket. Then by counting all the chips, the child can find the total cost. Direct modeling is used by young students to solve simple addition and subtraction problems and even some surprisingly difficult multiplication and division problems (Carpenter et al. 1993).

Direct modeling can be rather inefficient, however, especially for problems with larger numbers. Eventually, direct-modeling strategies are supplanted by oral or mental counting strategies. A large number of such strategies for addition have been identified (Resnick 1983; Carpenter and Moser 1984; Baroody and Ginsburg 1986; Siegler and Jenkins 1989). Two common strategies are "counting all" and "counting on from the larger addend."

A significant feature of most counting strategies is that the child must keep track of how many numbers have been said. To solve 3 + 4 by counting all, the child first counts three numbers (1, 2, 3) and then four more numbers (4, 5, 6, 7). This "double counting" can be tricky; using fingers or objects can help. Similar counting strategies exist for subtraction, such as counting up from the smaller number to the larger and counting back from the larger number.

How best to help children advance to more efficient strategies is an open question. Certainly laying out a strict sequence of strategies and expecting all children to adhere to it would be ill-advised. Not only do different children progress at different rates, but the same child may use different strategies on different problems or even on the same problem in different contexts. However, if teachers hesitate too much to demonstrate better methods, students' progress may be impeded.

One approach to encourage more efficient methods is to ask children to share their strategies. This method helps them improve their communication skills and learn from one another. **Figure 1,** for example, shows how two first graders used a hundreds chart to find 5 + 9. In a typical class, children will use and describe various approaches, so most children will encounter new but understandable techniques. The teacher may also propose and model strategies, taking care that certain strategies do not become "official" while other strategies are discouraged. The teacher should not be disappointed when a child does not adopt more efficient strategies right away—development may be advancing below the surface at the rate best suited to the child.

To encourage more efficient methods, ask children to share strategies

Class discussion of strategies should be supplemented with exercises designed to facilitate more sophisticated strategies. For example, the advance from counting all to counting on depends in part on skill in counting ahead the correct number of counts from another number. By practicing counting outside any problem context, children can develop competencies that support more sophisticated problem-solving strategies. A variety of such exercises should be included: counting forward and backward by 1's, starting at various numbers; skip counting, especially by 2's, 5's, and 10's; and counting forward and backward a given number of numbers: "Start at 8 and count forward 3" or "Start at 11 and count back 2," and so on.

Parts and wholes

Another central understanding that young children bring to school is that a quantity can be broken into parts that taken together equal the original quantity. They also understand that if they have some and get more, then they end up with more; and if they have some and lose some, then they end up with less (Resnick, Lesgold, and Bill 1990). Developing these basic "parts and whole" ideas further is essential to understanding addition and subtraction.

Ten-frames, like those in **figure 2,** are good for developing part-whole understandings involving the landmark numbers 5 and 10 (Thompson and Van de Walle 1984; Thornton and Smith 1988; Van de Walle 1994). These understandings are especially useful in addition- and subtraction-fact work. For example, the ten-frame for 8 in **figure 2** shows that 8 is 3 more than 5 and also 2 less than 10. The ten-frame for 4 shows that 4 is 1 less than 5 and 6 less than 10. Once students learn facts involving 5 and 10, especially the pairs of numbers that sum to 10, they can use their knowledge to solve other basic-fact problems.

Derived facts

Although most young children do not have automatic command of the basic facts, most adults do. In between is a stage in which some facts are known and others are not. During this stage, many children use the facts that they know to derive the facts that they do not know. Class discussion of such derived-fact strategies helps students learn from their peers and also legitimizes the use of strategies, thus encouraging the invention of further strategies (Steinberg 1985). Class discussion should examine the relative advantages of different strategies for various problems (Thornton and Smith 1988). Encouraging the discussion of multiple solutions enhances strategy and fact knowledge and helps students develop methods for mental and multidigit computation. Instruction to facilitate specific strategies can also be worthwhile.

The "doubles" facts are often useful for deriving unknown facts. For example, a child might solve 3 + 4 by noting that 3 + 3 = 6, so 3 + 4 must be 1 more than 6. Facts like 8 + 6 can be solved either by "sharing" (8 + 6 = 7 + 7 = 14) or by using a double and adding 2 more (8 + 6 = 6 + 6 + 2 = 12 + 2 = 14). Since doubles-based strategies are common, care should be taken that children learn the doubles facts early. Many games can be modified so that they involve doubles. For example, games with two dice can be played with one die doubled instead. A chart with examples of addition doubles, such as 6 eggs + 6 eggs = 12 eggs, can be kept as a class project and explored for patterns, for example, all the sums are even. Brief oral drills are also appropriate as children are consolidating their knowledge of these facts.

Many other common strategies involve 10. For example, a child using 10 might solve 9 + 7 in several steps: 9 + 1 = 10 and 10 + 6 = 16, so 9 + (1 + 6) = 16. To support such strategies, early attention should be given to complements of 10, such as 6 + 4, 7 + 3, and so on. The ten-frame activities described previously are ideal.

Children also use derived-fact strategies for subtraction. Some subtraction strategies are refinements in counting, such as using 10 as a bridge in counting up or down. For example, to solve 13 – 6, count up 4 from 6 to 10, and then up 3 more from 10 to 13, for a total counted of 4 + 3 = 7. Other strategies involve using known addition facts to derive unknown subtraction facts: 15 – 8 = 7, since 7 + 8 = 15.

Practice

The place of practice in school mathematics is much disputed. We think that a reasonable position was described by William Brownell more than fifty years ago. Brownell and his student Charlotte Chazal found that under certain conditions, practice can be harmful. Premature demands for speed, for example, caused many children simply to become quicker at immature approaches. Delaying drill was found to result in better understanding and ultimately in less need for drill (Brownell and Chazal 1935). Over the years, unfortunately, some educators have misunderstood this and similar research and have concluded that all practice is bad.

We believe that the right conclusion is that premature practice can be detrimental but that properly managed practice is essential in the development of expertise—whether the subject is piano, tennis, or the basic facts (Brownell 1956; Chase and Chi 1981; Siegler 1988; Anderson, Reder, and Simon 1996). Brief, engaging, and purposeful practice distributed over time is usually most effective. Problem solving is one important source for such practice, but games, computers, or even old-fashioned technology like flash cards and choral drills can also be useful.

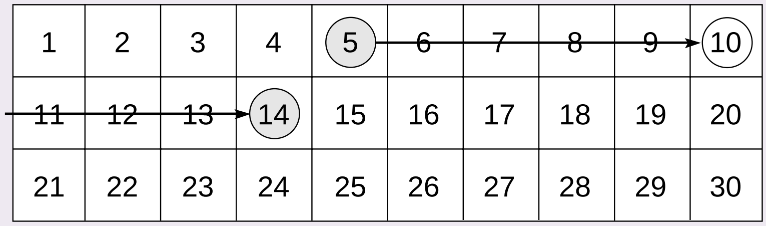
An instructional sequence

The preceding ideas can be used to sketch a possible instructional sequence for the addition and subtraction facts. Note that in this sequence, the facts are grouped by strategy rather than by sum. A double like 6 + 6, for example, may be easier than a problem like 4 + 3 and, accordingly, appears earlier in this sequence.

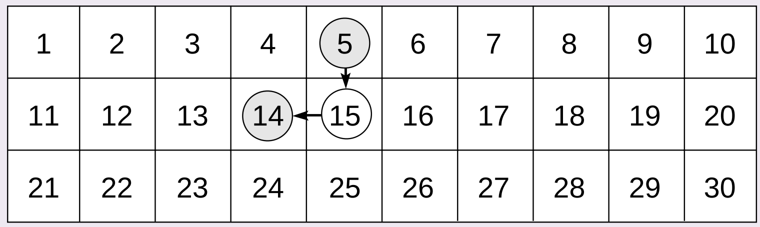
1. Basic concepts of addition; direct modeling and "counting all" for addition
2. The 0 and 1 addition facts; "counting on"; adding 2
3. Doubles (6 + 6, 8 + 8, etc.)

FIGURE 1

Hundred-chart strategies for 9 + 5



"I started at 5. Then I counted 5 more to 10, then 4 more to 14."



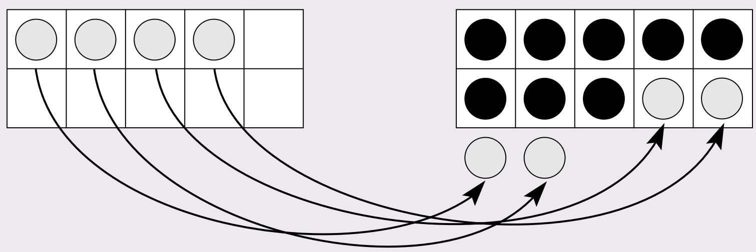
"I started at 5 and jumped ahead 10 to 15. But it was only 9, so I moved back 1 to 14."

FIGURE 2

Ten-frames showing that 4 + 8 = 12

4 + 8 = _____

Maria made 4 and 8 on the ten-frames. Then she moved 2 counters to make 10 and 2 more to make 12. So 4 + 8 = 12.



4. Complements of 10 (9 + 1, 8 + 2, etc.)
5. Basic concepts of subtraction; direct modeling for subtraction
6. Easy subtraction facts (– 0, – 1, and – 2 facts); "counting back" to subtract
7. Harder addition facts; derived-fact strategies for addition (near doubles, over-10 facts)
8. "Counting up" to subtract
9. Harder subtraction facts; derived-fact strategies for subtraction (using addition facts, over-10 facts)

How Can Fact Knowledge Be Assessed?

The assessment of children's fact knowledge should be balanced, based on multiple indicators, and aligned with instruction. Assessment should help the teacher evaluate not only answers but also how students are getting those answers and whether students understand the underlying mathematical concepts and connections. For example, a student might appear to know the basic facts during problem-solving activities but actually be relying on counting. Another student might be quite proficient on isolated facts but have a weak grasp of the concepts of the operations. A combination of assessment techniques can clarify each student's strengths and weaknesses and can help the teacher plan instruction.

What is meant by fact proficiency differs by age

What is meant by fact proficiency differs by age. For example, the first grader in **figure 3** used finger counting, doubles, and recall in answering various facts. These responses show a good range of mathematical understanding and indicate that the student is reasonably proficient in the basic addition facts. By third or fourth grade, however, we would expect all addition facts to be answered quickly by recall or automatic strategies.

Samples of students' work

Collecting samples of student work is a good way to gather evidence about students' knowledge and application of facts. These performance-based samples should come from activities in which students use facts. For example, **figure 4** shows a number-collection box in which a second grader recorded different ways of making a target number, in this example, 9. Such exercises help children develop their understanding of addition and subtraction and also afford opportunities for assessing fact knowledge.

Although performance-based samples offer evidence of conceptual understanding and applications, information about students' level of proficiency is often limited. Typically, for example, work samples do not reveal whether the student used counting, derived-fact strategies, or recall. The information is also limited to the particular numbers involved in the sample. Without additional information, a teacher might find it difficult to plan meaningful instruction.

Observations, class discussion, and interviews

Observing students engaged in games and problem-solving activities can yield rich information about their fact knowledge. For example, as students play a game, a teacher may notice whether they are using counting strategies, derived-fact strategies, or known facts. More important, the teacher can get a better idea of the range of students' knowledge with individual facts. Brief observational notes can help with planning individual and whole-class instruction: Tomás is still counting on, even for the easy facts; Juanita knows most of the double facts, and she also uses these facts to solve some of the near doubles. Useful information can also be obtained during class discussions as individuals explain their solutions to story problems or other problem-solving activities.

Short individual interviews are probably the best way to get a full picture of a student's progress with basic facts. Although these interviews are time-consuming, with a little planning a teacher can manage a five-minute interview twice a year with each student, perhaps spacing the interviews over a month. More frequent interviews with students who are having difficulties can help pinpoint problems.

Inventory tests

Although clearly an overreliance on timed tests is more harmful than beneficial (Burns 1995), this fact has sometimes been misinterpreted as meaning that they should never be used. On the contrary, if we wish to assess fact proficiency, time is important. Timed tests also serve the important purpose of communicating to students and parents that basic-fact proficiency is an explicit goal of the mathematics program. However, daily, or even weekly or monthly, timed tests are unnecessary.

An inventory test on all the addition and subtraction facts might be done at the beginning of second and third grades. These tests establish a baseline for measuring progress and provide information that can be useful in planning instruction. End-of-the-year tests, and perhaps mid-year tests, can be used to document progress. Similar inventories for multiplication and division facts might be given in fourth and fifth grades. We recommend

FIGURE 3

A “reasonably proficient” first-grade student

Student: [Student reads] Three plus five equals . . . . Hm. [Pause] Three. [Student then counts on fingers, putting up five fingers at one time.] Four, five six, seven, eight. Eight.
Teacher: How did you figure that out?
Student: I did it on my fingers.
[Child is shown card with 5 + 5 on it.]
Student: [Rapidly] Five plus five equals ten.
Teacher: How did you get that?
Student: I figured it out in my mind.
Teacher: You always knew it? [Student indicates yes.] Okay, what’s six plus six?
Student: [Fairly rapidly] Thirteen.
Teacher: How did you get that?
Student: Because I counted five and then added two more: five plus five and two more.
Teacher: And you got what, thirteen?
Student: Yeah. [Child reads next problem] “Seven plus nine equals. . . . ” [Pause. Then child begins to count on fingers. First, child apparently begins to count all—to seven on one hand. Then starts over, saying seven and starting over on fingers, putting up nine fingers one at a time.] Eight, nine, ten, . . . , sixteen. Sixteen.
Teacher: Sixteen. Okay, so here’s another one. If seven plus nine is sixteen, what’s nine plus seven?
Student: [Two seconds, then child responds with enthusiasm.] Sixteen!
Teacher: How do you know that?
Student: It doesn’t matter which one’s first. But they’re always, . . . they’re just always like, . . . no matter what is first, they’re always the same number.
Teacher: Here’s another one. Four plus ten.
Student: [Quickly] Four plus ten is fourteen.
Teacher: How did you get that?
Student: Well, I just figured it out.
Teacher: On your fingers.
Student: No. I thought.

against any timed tests during first grade, or any frequent use in the primary grades, because they work against a strategies approach to the facts. That is, in a timed situation, students will be less likely to explore the more sophisticated strategies necessary to make progress.

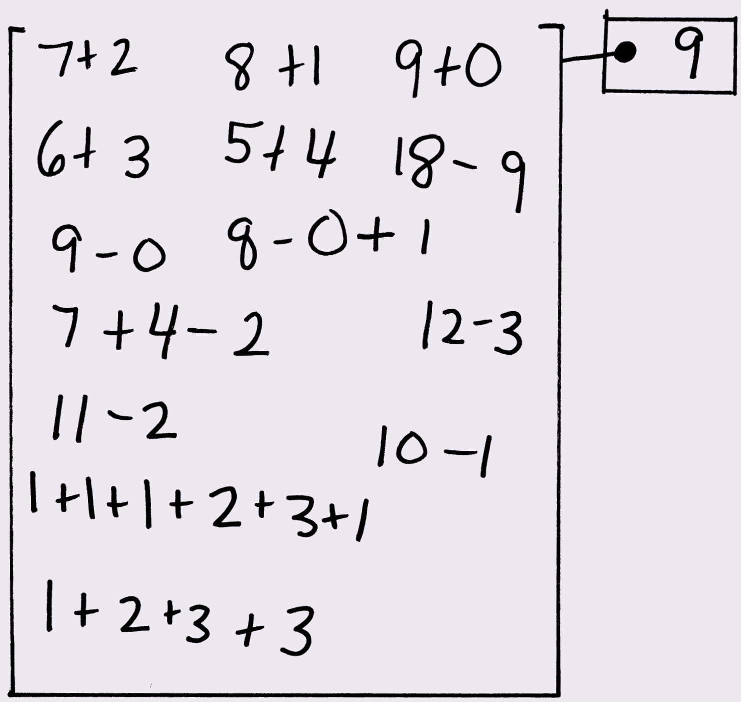
Small-scale diagnostic tests

Although positive inventory-test results are reassuring, they yield limited information. It may be, for example, that a student is proficient at some facts but uses counting for other facts. For this reason, it is helpful to test smaller sets of facts with short diagnostic tests linked to specific strategies. For example, after students work on the doubles, a quick test of the doubles facts can indicate whether students are ready to move on. As students move toward proficiency, short tests of mixed fact strategies—doubles, near doubles, and complements of 10—can also be useful for diagnostic purposes. A three-second rule is often used as a benchmark of automaticity (Van de Walle 1994), although some teachers prefer two seconds (Thornton 1990). Note that these criteria allow enough time for students to use efficient strategies or rules for some facts.

If we expect students to move from counting

FIGURE 4

“Number-collection box” for 9



strategies toward facility with facts, then an occasional low-stress test or practice is consistent with our goals and the message that we want students to receive. The crucial point is to emphasize individual progress. In kindergarten and first grade, counting strategies are appropriate for solving the basic addition facts, but we should have concerns about students who are still "counting all" in the middle of second grade or who have not mastered even the easiest addition facts. Not diagnosing these students' difficulties and planning appropriate instruction for them does them a disservice. A balanced approach to assessment—work samples, some observations, some test information, and some interview information—gives the teacher, the student, and the parent a more complete portrait of the child's fact knowledge, how it is connected to other mathematical knowledge, and what progress is being made.

Neglecting basic facts may undermine reforms now under way

Conclusion

A strategies-based approach to the basic facts has several advantages. First of all, it works: children do learn their facts. Rathmell (1978) found that teaching children thinking strategies facilitates their learning and retention of basic facts. More recent studies have confirmed this effect again and again. These findings should not be surprising: a strategies approach helps students organize the facts in a meaningful network so that they are more easily remembered and accessed. Further, although many facts become automatic, adults also use strategies and rules for certain facts. Many strategies, such as properties of the multiples of 9, both support facts and supply links to other mathematical concepts, such as divisibility. Many researchers have recommended strategies-based approaches for learning the basic facts, including Thornton (1978, 1990), Cook and Dossey (1982), Myren (1996), and Chambers (1996).

A strategies-based approach also builds students' understanding and confidence. De-emphasizing rote memorization encourages students to use their common sense in mathematics, thus supporting concept development. International research confirms that early fact automaticity and problem solving are not discrepant goals (Fuson, Stigler, and Bartsch 1988; Stigler, Lee, and Stevenson 1990). The cost in instructional time is also low: delayed practice often means less practice. Children's success at learning their facts also reassures parents about their children's mathematics program.

Certain pitfalls must be avoided in a strategies-based approach. One danger is that children might learn strategies by rote, so that mindless memorization is replaced by equally mindless "strategies" (Cobb 1985). Another possibility is that class discussion might degenerate into the tedious recitation of every imaginable method, with little critical appraisal of the various approaches. Encouraging multiple ways to solve fact problems may also lead students to conclude that memorizing the facts is not important. We believe, however, that in most situations a thoughtful and sensitive teacher can avoid these hazards.

Our purpose has been to address important questions about the basic facts, for fear that neglecting them will undermine reforms now under way. We worry that our efforts to correct for a narrow focus on lower-level skills will lead to an overcorrection. We recall Brownell's warning at the beginning of the New Math era: "In objecting to the emphasis on drill prevalent not so long ago, we may have failed to point out that practice for proficiency in skills has its place too" (1956). We must remember that successful education involves both basic skills and higher-order processes.

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