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Elementary Preservice Teachers' Changing Beliefs and Instructional Use of Children's Mathematical Thinking

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In this research, we examined changes in preservice elementary school teachers' beliefs about teaching and learning mathematics and their abilities to provide mathematics instruction that was based on children's thinking. The 34 participants in this study were introduced to Cognitively Guided Instruction (CGI) as part of a mathematics methods course. Belief-scale scores indicated that significant changes in their beliefs and perceptions about mathematics instruction occurred across the 2-year sequence of professional course work and student teaching during their undergraduate program but that their use of knowledge of children's mathematical thinking during instructional planning and teaching was limited. Preservice teachers may acknowledge the tenets of CGI and yet be unable to use them in their teaching. The results raise several questions about factors that may influence success in planning instruction on the basis of children's thinking.

Key Words: Children's strategies; Constructivism; Early childhood, K-4; Pedagogical knowledge; Planning, decision making; Preservice teacher education; Teacher beliefs

This study was designed and carried out as an attempt to document the effect of introducing preservice elementary school teachers to Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Peterson, & Carey, 1988). CGI's effectiveness in changing teachers' beliefs about mathematics instruction and the nature of mathematics instruction in primary grades is well documented (Fennema et al., 1996; Fennema, Franke, Carpenter, & Carey, 1993; Peterson, Fennema, Carpenter, & Loef, 1989). Teachers prepared in CGI spend more time having their students solve problems, listen more to their students, and are more likely to expect students to find multiple solution strategies to problems than teachers who are not prepared in CGI (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989). CGI also results in improved performance by primary-grade students on both standardized and problem-solving tests (Carpenter et al., 1989; Fennema, Carpenter, & Peterson, 1989; Peterson et al., 1989). In question is whether similar findings would accrue to the integration of CGI within preservice teacher education programs.

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The research reported in this article was part of a larger project, the Primary Preservice Teacher Preparation Project (funded by the National Science Foundation) that was designed to begin to investigate the effects of including information about CGI in preservice teacher education programs. The project was conducted through the University of Wisconsin and involved preservice teacher education programs at three sites. The data reported here are from the project site at the University of North Carolina at Greensboro. Specifically, we examined (a) preservice teachers' beliefs about mathematics instruction and (b) mathematics instruction provided by two of these preservice teachers. In our analysis of mathematics instruction we focused on how the two preservice teachers used knowledge of their students' mathematical thinking in instruction during the student-teaching semester.

BACKGROUND

Cognitively Guided Instruction

CGI is an approach to helping "teachers use knowledge from cognitive science to make their own instructional decisions" (Carpenter & Fennema, 1991, p. 10). Children's knowledge and the teacher's understanding of that knowledge are central to instructional decision making. Teachers plan instruction using research-based knowledge about children's mathematical thinking and well-defined taxonomies of problem types and children's solution strategies for arithmetic operations (Carpenter & Fennema, 1991; Carpenter & Moser, 1983). Teachers seek specific information about individual students' thinking and understanding and then adjust the level of content to match individual students' performance levels.

A single model of a "CGI teacher" does not exist. Instead, teachers use CGI in a manner that fits their own teaching styles, knowledge bases, and beliefs, as well as the needs of their students. Similarities do exist, however, across "CGI classrooms." For example, students in CGI classrooms spend most of their mathematics instruction time solving various problems by creating their own solutions instead of by following a set of procedures provided by an outside source such as the teacher or the mathematics textbook. Students also spend a considerable amount of time sharing their solution strategies and asking questions of one another and the teacher until they have developed an understanding of the problem solutions.

Teachers who use CGI principles when teaching (a) believe that their understanding of children's thinking is a critical component of instructional planning, (b) facilitate children's problem solving and discussions of children's thinking, (c) listen to their children and question them until the students' thinking becomes clearer, and (d) are willing and able to make instructional decisions that are appropriate to the mathematical needs of their students (Fennema et al., 1996). As a result, significant positive correlations exist between CGI and students' mathematics problem-solving achievement (Peterson et al., 1989), ability to solve complex addition and subtraction word problems (Fennema et al., 1989), and level of recall of number facts (Carpenter et al., 1989).

Experienced teachers are able to apply the research-based knowledge that they gain while learning about CGI to an already existing set of understandings about children's thinking and about their own preferred teaching styles. In contrast, preservice teachers are likely to have limited knowledge about children's mathematical thinking and to be in the process of developing preferred styles of teaching; indeed, their teaching styles may be shifting repeatedly while they gain experience and pedagogical content knowledge (Shulman, 1986). Thus, their preparation in CGI may not be synthesized and applied in a manner similar to that of experienced teachers, and the extent to which they consider CGI principles in instruction may be significantly different from that of experienced teachers.

Beliefs About Mathematics Instruction

Teachers' beliefs about teaching and learning mathematics significantly affect the form and type of instruction they deliver (Clark & Peterson, 1986; Richardson, Anders, Tidwell, & Lloyd, 1991). If teachers' beliefs are compatible with the underlying philosophy and materials of a curriculum, there is greater likelihood that the curriculum will be fully implemented (Hollingsworth, 1989; Richardson, 1990). These findings are supported in the CGI literature. A critical factor in a teacher's use of CGI principles is his or her beliefs about teaching and learning mathematics (Fennema et al., 1996; Fennema et al., 1993; Peterson et al., 1989). Preparation in CGI helps primary teachers organize and expand their knowledge about children's thinking while they construct instructional strategies on the basis of what they are learning about their students' thinking (Fennema et al., 1996; Fennema et al., 1993). Thus, the processes of learning about research on children's mathematical thinking and using that knowledge while interacting with students are associated with changes in both teachers' beliefs and the type of instruction they provide their students. Precisely how these findings apply to preservice teachers is unclear.

Preservice teachers' general beliefs about teaching are tenacious (Holt-Reynolds, 1992) as are their beliefs about teaching and learning mathematics (Ball, 1989; McDiarmid, 1990). Learning new theories and concepts may have little effect in changing preservice teachers' general beliefs about teaching practices (Calderhead & Robson, 1991; Kagan, 1992). Instead, preservice teachers' beliefs seem to be drawn from previous vivid episodes or events in their lives (Pajares, 1992); their beliefs about teaching and learning appear to be generalizations derived from their own experiences as students (Holt-Reynolds, 1992; Knowles & Holt-Reynolds, 1991). Posner, Strike, Hewson, and Gertzog (1982) suggested that for existing beliefs to be replaced or reorganized, new beliefs need to be intelligible and appear plausible. For example, the framework underlying the content presented in mathematics methods courses needs to be consistent with the framework of the mathematics education program that preservice teachers observe and implement during field experiences. If the two frameworks are in conflict, the theories and concepts presented during the mathematics methods course may not seem plausible and may be rejected.

Schram, Wilcox, Lanier, and Lappan (1988) found that preservice teachers' beliefs about what it means to know mathematics were challenged when conceptual development, group work, and problem-solving activities were emphasized during a mathematics content course. However, emphasizing these components had little effect on the preservice teachers' beliefs about what should be included in elementary school mathematics education. Schram and Wilcox (1988) concluded that instead of changing beliefs, some preservice teachers fit existing beliefs to their new experiences. These conclusions were supported by McDiarmid (1990), who indicated that many preservice teachers resisted change even when a course was designed specifically to challenge their underlying beliefs about mathematics education. Despite their experiences in the course, most of the preservice teachers in his study ended the course still believing that a teacher's role is to explain the answer instead of to help students develop understanding.

It appears that even full-time teaching during a teacher preparation program may not be a powerful change agent inasmuch as preservice teachers' beliefs remain stable across the student-teaching experience (Calderhead & Robson, 1991; McDaniel, 1991; McLaughlin, 1991). Zeichner and Liston (1987) found that instead of changing beliefs, preservice teachers became more skillful in expressing and implementing their points of view. These studies, however, focused on beliefs about teaching and learning, in general. Whether their results generalize to preservice teachers' beliefs about teaching and learning mathematics is unclear.

Brousseau and Freeman (1988) indicated that teacher preparation programs generally do not challenge students' initial beliefs about mathematics education. As a result, preservice teachers may conclude their programs of study without examining their own perspectives about teaching and learning mathematics. Kagan (1992), on the basis of a review of 40 learning-to-teach studies conducted between 1987 and 1991, identified three elements that seem essential for changing preservice teachers' beliefs. First, preservice teachers need to have extended opportunities to interact with and study students. Second, the content of their university courses needs to be connected to the exigencies of classroom teaching; university courses need to focus on procedural knowledge and practical strategies as well as theory. Third, their field experiences need to include opportunities to work with classroom teachers who engage in ongoing self-reflection by questioning and reconstructing their own pedagogical beliefs. As discussed later, the first two elements were included in the present study as part of the teacher preparation program. The classroom teachers (i.e., the on-site teacher educators) who supervised the field experiences of the participants in this study may have engaged in self-reflection practices, but it was not part of the criteria for their selection as field-experience supervisors.

Teacher Preparation Program

In our elementary education teacher preparation program, students are required to complete 46 semester hours of liberal arts courses that include 6 hours of mathematics course work and to complete a second major that consists of a minimum of 24 semester hours of course work in one of the arts or sciences. The program incorporates professional development schools (PDSs) that support sustained experiences in classrooms to help preservice teachers integrate what they are learning about teaching (i.e., theoretical frameworks) with what they are observing, doing, and experiencing in classrooms (i.e., practice). As a result of the partnership between the university and PDSs, classroom teachers serve as on-site teacher educators. They meet with university faculty to plan field experiences for the preservice teachers and sometimes model instructional activities as part of the methods courses. They typically are willing for undergraduates to try out various instructional methods during the field experiences.

Preservice teachers take all their professional courses in cohort groups beginning in the junior year. The sequence of professional course work includes a mathematics methods course taught during the fall semester of the senior year. Preservice teachers also complete 10 hours per week of internship in the PDSs during both semesters of the junior year and the fall semester of the senior year. Full-time student teaching is completed during the spring semester of the senior year in the same classroom in which the senior fall-semester internship is completed.

METHOD

Participants

Thirty-four members of an undergraduate cohort of preservice teachers took part in the study. At the beginning of the study, they were commencing their 2-year sequence of professional course work in elementary education.

Only two of the on-site teacher educators working with this cohort were experienced CGI teachers, and both taught at the same PDS. One was a third-grade teacher and the other taught kindergarten. Thus, only two preservice teachers in the cohort completed their senior-year field experiences in classrooms of experienced CGI teachers.

Because this study was undertaken to document changes in preservice elementary school teachers' beliefs about teaching and learning mathematics, we wanted to monitor changes in their instruction. Two preservice teachers (Helen and Andrea) were selected as cases for in-depth study because of similarities in their senior-year field experiences. They completed their senior-year internships and student teaching in adjacent third-grade classrooms. Thus, they shared a common grade-level curriculum, worked with the same school personnel outside the classroom (e.g., administrators, resource teachers), and were not in PDSs that might have had different school philosophies. Helen's on-site teacher educator was a third-grade teacher with extensive experience with CGI. In contrast, the CGI experience of Andrea's on-site teacher educator was limited to participation in a 2-hour "awareness" workshop about CGI. Other differences existed between the two preservice teachers: Their junior field experiences were in different schools and at different grade levels, and Helen's second major was psychology whereas Andrea's was speech communication.

Cohort Leaders

The cohort was led by the first author, who is a faculty member in the Department of Curriculum and Instruction. She taught these students' mathematics methods course and conducted their weekly seminars during the three semesters of internship and student teaching. An experienced classroom teacher, who was a full-time graduate student in the same department, assisted with the leadership of the cohort. These two leaders served as the liaison between the university and the PDSs and also worked collaboratively with the on-site teacher educators in supervising field experiences and student teaching.

The university faculty member, who was an experienced classroom teacher, had been prepared in CGI through professional development workshops at the University of Wisconsin. Also, she spent a considerable amount of time each year working with children and observing in experienced CGI teachers' classrooms. The graduate student participated in one of the workshops at the University of Wisconsin during the summer prior to the cohort's mathematics methods course, and she was an observer during the mathematics methods course.

Mathematics Methods Course

The 3-semester-hour mathematics methods course met once a week, for 2 hours and 50 minutes per session, during the fall semester of the students' senior year. Course content centered around process learning and the national curriculum reform; problem solving, communicating mathematically, reasoning, and making mathematical connections were emphasized. Course requirements were designed to provide opportunities for preservice teachers to focus on children's thinking and included (a) conducting two case studies (with assessment interviews) of students in the internship classroom; (b) planning, implementing, and evaluating a mathematics lesson in the internship classroom; and (c) carrying out observations of three mathematics lessons taught by the on-site teacher educator. In conducting their case studies, the preservice teachers had opportunities to focus on the thinking and understanding of individual students. Teaching the mathematics lesson helped the preservice teachers learn to monitor children's thinking during whole-class instruction. During classroom observations the preservice teachers could focus on different aspects of the teacher's role as it relates to understanding children's thinking: the amount of wait time used by the on-site teacher educator when she questioned students, the types of questions (i.e., factual, open-ended, restated, and probing) the teacher asked along with the responses of the students, and the instructional procedures the teacher employed during the lesson.

CGI was introduced through a five-session module. We introduced problem types for the basic operations and children's solution strategies (Carpenter, Fennema, & Franke, 1993) during the first four sessions, and knowledge of children's geometrical thinking (Lehrer, Fennema, Carpenter, & Osana, 1992) was addressed during the fifth session. In general, to introduce problem types and solution strategies, we presented a mathematics story problem and asked the preservice teachers to find alternative solutions to the problem. After sharing some solution strategies, the preservice teachers viewed videotaped examples of children's solutions to the same problem and discussed how their solutions were similar to or different from those of the children. We also focused the discussion on what problems might be given next to an individual child, thus encouraging the preservice teachers to begin using knowledge of children's thinking to plan instruction. During the fifth session, in a PDS second-grade classroom, the instructor conducted a demonstration geometry lesson that focused on the children's visual, descriptive, and relational thinking about shapes. The discussion that followed this lesson centered around the information gained or not gained about the students' thinking and modifications in the lesson that would have provided additional information about the students' understanding. Immediately following this discussion, the preservice teachers, individually or in pairs, used the same instructional activities with a student from another second-grade classroom. The session concluded with the preservice teachers sharing what they learned or did not learn about their students' geometrical thinking, with possible reasons for their outcomes.

Instrumentation

To assess changes in the preservice teachers' beliefs about teaching and learning mathematics, we administered the CGI Belief Scale (Peterson et al., 1989) four times: beginning of the professional preparation program (i.e., start of the fall semester of the junior year), beginning of the mathematics methods course (i.e., start of the fall semester of the senior year), beginning of student teaching (i.e., start of the spring semester of the senior year), and end of student teaching (i.e., end of senior year).

The Belief Scale consists of 48 items designed to assess teachers' beliefs, which are categorized on four subscales: Role of the Learner, Relationship Between Skills and Understanding, Sequencing of Topics, and Role of the Teacher. Respondents rate each item using a 5-point Likert scale of *strongly agree, agree, undecided, disagree,* or *strongly disagree*. Each subscale measures interrelated but separate constructs. High scores on the Role of Learner subscale indicate a belief that children, instead of being receivers of knowledge, are able to construct their own knowledge. High scores on the Relationship Between Skills and Understanding subscale indicate the belief that skills should be taught in relationship to understanding and problem solving rather than in isolation. High scores on the Sequencing of Topics subscale indicate a belief that the sequencing of topics for instruction should be based on children's natural development of mathematical ideas rather than on the logical structure of formal mathematics. High scores on the Role of Teacher subscale indicate a belief that mathematics instruction should facilitate children's construction of knowledge rather than consist of the teacher's presentation of knowledge. Peterson et al. (1989)

reported that internal consistency estimates for each subscale ranged from .57 to .86; internal consistency of teachers' scores on the total belief scale was .93.

In addition to the data from eight on-site formal observations of each prospective teacher (two by each cohort leader and four by the on-site teacher), data for the more in-depth study of Helen and Andrea included reflective journal entries during the mathematics methods course and student teaching, four videotaped mathematics lessons during the student-teaching semester, and three open-ended interviews. One interview was conducted during the fall semester of the junior year, one during the fall semester of the senior year, and one at the end of student teaching. The interviews were planned by the authors in collaboration with a third departmental faculty member who also had been prepared in CGI. Graduate students conducted the first two interviews as part of the requirements for a component on ways of knowing in a human development course that was taught by the third departmental faculty member. The final interview was conducted by a doctoral student who was paid to conduct the interviews. To eliminate any effect due to an interviewer's leading the interviewees' responses in a given direction, we chose interviewers who were not knowledgeable about CGI. Each interview focused on the teacher's role in mathematics education (e.g., How do you figure out what children know in mathematics?); the final interview also addressed decisions that each participant had made while teaching a lesson that the interviewer had observed (e.g., What, if anything, happened during the lesson today that caused you to change your plans for the lesson?).

RESULTS

We present the findings of this study in two parts. Results concerning the preservice teachers' beliefs are presented first followed by the results related to Helen's and Andrea's beliefs and their use of CGI-based knowledge in their teaching.

Preservice Teachers' Beliefs

The 34 preservice teachers' mean scores on the Belief Scale across the four administrations are given in Table 1. Changes in the mean total scores during consecutive administrations of the Belief Scale were 2.3, 24.7, and 12.2, respectively. The preservice teachers' Beliefs Scale scores changed little during the first year of their program, increased significantly during the mathematics methods course, and continued to increase significantly across the student-teaching experience. A repeated-measures analysis of variance and follow-up paired *t*-tests of scores at adjacent times showed a significant overall time effect ($p < .0001$) for all four subscales as determined by Wilks's Lambda and its associated *F*-statistic. Subscale means changed little across the first year of the study, but there were significant increases ($p < .0001$) in the means of all four subscales during the mathematics methods course. Also, on average, the preservice teachers' belief scores continued to increase significantly ($p < .005$) during student teaching, with the greatest increase occurring in their beliefs about sequencing of topics.

Table 1
Mean Scores (and Standard Deviations) on the Belief Scale Across Administrations for the 34 Preservice Teachers

Subscales	Beginning of program	Beginning of methods	Beginning of student teaching	End of student teaching
Role of Learner	36.5 (5.2)	35.7 (6.0)	42.2*** (6.7)	44.9* (5.8)
Skills and Understanding	36.9 (4.7)	37.9 (5.1)	44.7*** (6.6)	48.0* (7.4)
Sequence of Topics	39.3 (4.6)	40.3 (4.9)	45.3*** (6.1)	48.7** (5.0)
Role of Teacher	38.5 (5.0)	39.4 (4.8)	46.0*** (6.2)	48.8* (5.3)
Total beliefs	151.2 (14.8)	153.5 (17.5)	178.2*** (21.7)	190.4* (20.4)

Note. The maximum score is 60 for each subscale and 240 for the total score.
*Change from previous mean significant at $p < .005$. **Change from previous mean significant at $p < .0005$. ***Change from previous mean significant at $p < .0001$.

Helen's and Andrea's Beliefs and Their Use of CGI Principles

When we analyzed the data concerning Helen and Andrea, we found differences in (a) the ways their beliefs, as measured by the Belief Scale, changed across their preparation program and (b) their use of CGI principles during mathematics instruction.

Belief changes. Helen's and Andrea's respective scores on the four administrations of the Belief Scale are presented in Table 2. Although belief scores of both participants increased across the 2 years, the changes varied by preservice teacher. Helen's overall belief scores about teaching and learning mathematics increased substantially across the methods course and continued to increase during student teaching. In contrast, Andrea's overall belief scores increased considerably across the first three semesters of her preparation program, including substantial increases during the mathematics methods course, but little change occurred during the student-teaching experience.

Table 2
Helen's and Andrea's Subscale and Total Scores on the Belief Scale Across Administrations

Subscales	Beginning of program	Beginning of methods	Beginning of student teaching	End of student teaching
Helen				
Role of Learner	35	36	48	47
Skills and Understanding	37	42	52	58
Sequence of Topics	45	45	49	56
Role of Teacher	46	47	49	49
Total beliefs	163	170	198	210
Andrea				
Role of Learner	30	32	51	51
Skills and Understanding	34	42	48	52
Sequence of Topics	33	39	55	52
Role of Teacher	31	33	51	49
Total beliefs	128	146	205	204

Note. The maximum score is 60 for each subscale and 240 for the total score.

The two preservice teachers differed in their belief-score changes by subscale area. Their belief scores about the role of the learner during mathematics instruction increased somewhat similarly across the four administrations of the Belief Scale with the greatest change occurring during the mathematics methods course and no change occurring during student teaching. Their belief scores about mathematical skills and understanding increased continually across the program, but Helen's score changed most during the semester when she was enrolled in the mathematics methods course, whereas Andrea's seemed to change most because of her experiences during the first year of the program. Helen's belief scores about the sequencing of mathematics topics did not change during the first year of the program, increased somewhat after the mathematics methods course, and continued to increase during student teaching. Andrea's belief scores in this area increased somewhat during the first year and substantially during the mathematics methods course, but the scores decreased somewhat during student teaching. Helen began the program with a belief about the role of the teacher that tended more toward a perspective of the teacher's facilitating student learning, and Andrea began the program with a set of beliefs that tended more toward the teacher's telling students what they need to know. By the conclusion of the preparation program, however, Helen and Andrea shared similar perspectives about the teacher's role. Helen's belief scores related to the role of the teacher remained rather stable across the 2 years. In comparison, Andrea's belief scores about the role of the teacher increased considerably during the mathematics methods course but decreased slightly during student teaching.

Helen's use of CGI principles. Helen's reflective writings and interviews illustrate her beliefs across the program of study. Although Helen's score on the Belief Scale at the beginning of the study indicated a belief that the role of the teacher was to facilitate learning, she indicated in a written reflection paper at the beginning of the mathematics methods course that the teacher's role was to model problem solutions for students. During the mathematics-methods-course semester, she stated that a teacher should question children to find out what they were thinking as they solved problems. After Helen had gained instructional experience during student teaching, she questioned the use of worksheets and stated in a journal entry that teachers need to help students understand what they are learning. This reflection supports the changes that occurred in her beliefs about skills and understanding during student teaching. Although the most significant change in Helen's overall belief scores on the Belief Scale occurred during the mathematics-methods-course semester, her reflective journal entries and comments during interviews indicate that changes occurred across the entire 2 years:

> Children learn mathematics through modeling others. I think children need some guidance but then they go use their own "style." (*Journal entry, September, Junior Year*)

> [Teaching mathematics] means to look at how children are thinking and how they solve their mathematics problems … making information relevant to what the children are going to do later in life.… You ask questions to get them to share their pre-

> vious knowledge and you observe them and keep records.… Listening to children's conversation can tell you a lot. (*Journal entry, December, Junior Year*)
>
> This probably sounds horrible to some people, but I have not really had a chance to look through the math textbook [for this grade] until today (a teacher workday).… Regular "student pages" are almost identical to what I used in elementary school, boring, fill-in-blank pages.… "Fun stuff" was in the teachers' guide … fun, useful, and very "CGI-ish." It was almost as if they were saying, "OK, once your students *suffer* through these boring worksheets, then you can let them … try to understand it. (*Journal entry, February, Senior Year*)
>
> [The teacher's role] is to make the children want to learn and to facilitate their learning rather than telling them [what they need to know]. (*Interview, April, Senior Year*)

Helen appeared to believe in the principles of CGI, and she was fairly successful in applying some of these during student teaching. She planned and implemented instruction that was based on problem solving, and she facilitated student understanding and critical thinking through a rather high level of questioning. This finding is illustrated by the following excerpt from a lesson she taught in February, shortly after she assumed full-time responsibility for the class during student teaching. The third graders were solving the following problem: "Tyrone wants to buy seven baseball cards. Each card costs 5 cents. How much money does he need?" Prior to this segment in the lesson, one of the students in the class, Erik, had shared how he had solved the problem by counting out dimes and pennies: "I knew I could get 20 cents out of 4 nickels, so I had ten, twenty, thirty, and then I counted pennies, one, two, three, four, five to equal thirty-five." (A fictitious name has been assigned to each student.)

Helen: Who can show me a way to do it where you use all the same coins?

Alice: I started counting by fives and he wanted … to buy seven baseball cards and each cost 5 cents, so I got seven nickels and started counting.

Helen: Why did you use a nickel?

Alice: Because they were 5 cents each and I counted by fives.

Helen: How did you know when to stop [counting]?

Alice: When I ran out of nickels.

Helen: You counted out the seven nickels first?

Alice: Yes.

Helen: Okay, very good.… Who can tell me what is different between the way Erik did it and the way Alice did it?

Susan: Erik had more coins than Alice.

Helen: "Erik had more coins than Alice." Very good.… Who can tell me something alike about what they did? They did one thing exactly alike.… Can anyone come up [to the overhead projector] and write a number sentence for this story problem?

Helen's use of the teaching strategy of asking questions that facilitated and promoted children's critical thinking (e.g., asking students to compare different solution strategies) was evident across the student-teaching semester. She also asked probing questions to gain further information about students' solution strategies (e.g., "How did you know how many nickels to put down and how many pennies?", "Why did you use 20 children?", and "Would you say that the units or the longs would give you more of an exact answer? . . . Why the units?"). Yet, there were also occasions during some of Helen's lessons when she demonstrated behaviors that seem indicative of directed teaching. This fluctuation between facilitating and telling was evident during a lesson in mid-April, one week prior to the conclusion of her student-teaching experience.

Helen: What do you think this … means, *unit of measure*?

Jose: What you used to go around the book.

Helen: What about this word right here? That's one of your spelling words. Anybody remember how to say that word?

Bobbie: *Perimeter.*

Helen: Perimeter, and that's what you have been measuring.… That's the distance around something.… Who can tell me one unit of measure you used?

Erin: Those cubes.

Helen: Okay, I'm gonna put "long base 10s" (writes on chart). And how many of those did it take?

Erin: Ten.

Helen: So, we would say the perimeter of your book is 10 long base 10s. Did anyone use the long base 10s and get a different answer?

Michael: I got 11.

Helen: You got 11 (records number on chart). Who has an idea about why they think this is different?

After the preceding segment, students discussed reasons for getting different answers when they had used the same book and unit of measurement; Helen asked probing questions that resulted in the students' identification of differences in how the two students placed the base-ten longs on their respective books. Subsequently, the students discussed units that could be used to measure the perimeter of the room. Prior to the exchange excerpted below, Helen had indicated that everyone was going to use a piece of string and had asked how they would use it to determine the perimeter.

Deanna: You could take the strings and go around the room and then take the ruler to see how long each string was, so you'd know how long the string was to count how long they are.

Helen: Okay, to see how many inches or feet there are? … Okay, do we need to use the ceiling?

Deanna: No.

Helen: We can use what?

Tien: The floor.

Helen: The floor. Anywhere, really; you can use the wall. I think it would be easiest, well I don't know. It might be easier to use the wall. Whatever you want to use.… You all came up with some good ways to figure out the perimeter.… I'm going to give each two people a string … [and] assign you a wall.… So, if you had this wall, where are you going to start?… So one partner—I need a volunteer—will hold it there? So Sandy is going to hold it there and I'm going to bring it around here. How many strings is the wall so far?… Okay, you let go of your end, Sandy, and bring it around the wall. How many strings is that?

Although Deanna's response indicated a clear understanding of how the room's perimeter could be measured, it appears Helen believed that she needed to demonstrate the process before the students could proceed on their own. After the measurements for each wall were determined, the lesson was concluded with a rather rich student-centered discussion that focused on determining the perimeter of the room on the basis of the measurements obtained for each wall.

Helen: How can I tell what the perimeter of the room is?

Erin: Add all of those [measurements for each wall] up?

Helen: Why should I add all of those together?

Erin: 'Cause that's how, um, how, um, you know, um how long the walls are.

Helen: Let's see if we can do that. Can we round these up? Those that have a half? Think that would be okay?

Tien: Yeah. Two halves are a whole.

Helen: How did you know that?

Tien: Half of the string.…

Helen: (Interrupts Tien) Okay, this is half of it.

Tien: And half and half puts together a whole string.

Helen: Okay so if we put this half and this half, we have one whole?

Tien: Yes.

Helen: So how am I going to remember to do that?

Jeremy: Cross the halves out.

Helen: Cross the halves out and put a 1 (does this on overhead). Okay, that works for me. Does everyone understand that?… So our room is thirty-eight and a half what?

Students: Feet. Strings. Yards. (Shared simultaneously)

Helen: Strings. Those strings are close to a yard, but we're just going to say *strings*. And what's that called?

Students: Perimeter.

It is interesting to note that during this part of the lesson, which illustrates a focus on children's thinking, there surfaced occasionally shades of teacher-centeredness, illustrated by Helen's use of the pronoun *I* (referring to herself) and by her telling students that the unit of measurement is strings.

Andrea's use of CGI principles. As documented by the reflections presented below, when Andrea began her program of study, she believed that memorization of facts was the framework for learning mathematics. By the conclusion of the mathematics methods course, she indicated that children needed to have opportunities to discover mathematical concepts through explorations of different problems and to build on what they already know. She also indicated that asking students questions was more important than telling them what they need to know. This perspective remained stable across the student-teaching semester.

> Children learn mathematics through memorizing the facts and symbols of math. They then take these skills and really learn them through much practice. (*Journal entry, September, Junior Year*)

> [The teacher lets] the children … discover learning instead of getting up there and telling them what [they] are going to learn. [The teacher lets] them discover through their manipulatives or whatever … the concepts and build on what they already know instead of just … doing everything for them and telling them everything. [The teacher figures out what children know by letting] them have a chance to talk,… write, and … show you what they already can do. (*Interview, December, Junior Year*)

> I am realizing that my students perform better when given story problems than when they see the same problems written out in number-sentence formation. (*Journal entry, March, Senior Year*)

During an interview near the end of student teaching, Andrea stated that the most important thing she had learned about teaching mathematics was the importance of questioning and trying to find out what students were thinking. However, she did not seem to realize that for the purpose of making informed instructional decisions teachers have to interpret students' responses to understand what they know. Instead, she appeared to focus more on whether the students' answers matched the responses she was expecting. In a journal entry that she wrote following a lesson she had taught during her third week of full-time student teaching, Andrea reported that her "best" questions were those designed to get students to show their processes. The transcript of this lesson, however, showed that she moved away from students who gave wrong answers and followed up only with students who gave correct answers. At one point in this lesson, she asked her third-grade students, "How are the ways [that certain students solved the problem] different? … Could you tell anything about what the person was thinking?" Although responses to these high-level questions might have provided her with valuable information to use in planning future instruction, Andrea gave the students only a few seconds to think about or respond to the questions. She did not probe the two responses that she accepted, thus missing an opportunity to gain more in-depth knowledge of those students' understanding. This type of comparative questioning did not occur again during any of her videotaped lessons. Instead, she appeared to pursue only correct thinking or thinking that was aligned with a predetermined procedure that she wanted the students to learn. This practice is illustrated with the following segment from a March lesson, during which she seems to have had in mind a predetermined procedure for writing a fraction.

Andrea: What fraction of your M&Ms is purple?

Rashida: Well, I only had 14 M&Ms and only one was purple, so I had one fourteenth.

Andrea: One fourteenth. How did you get your 14? How did you know what went on the bottom?

Rashida: Because it's the number that you had, the denominator.

Andrea: It's the denominator and that is what again?

Rashida: The number of things that you have.

Andrea: Okay, and how did you all come up with your top number?

Latasha: You see how many you got in the purples?

Andrea: Of the purples. Okay.… Now I would like to know the fraction of pinks and yellows.

Kim: Five fifteenths.

Andrea: Now how did you get five fifteenths?

Kim: Because I counted both of these as 3 plus 2 equals 5 and then I counted these [everything except the pinks and yellows] and that was 15.

Andrea: Okay, when we do fractions, do we subtract the part … the yellows and pinks from the overall group number? How do we get that bottom number? Where does our number come from? We need to decide how we get that first.

Terry: From the amount, the number of everything.

Andrea: The number of every single one or just.…

Terry: The number of every one in the group that you're using.

Andrea: Okay, every one in the group that we're using. So, how many M&Ms do you have altogether, including every one of every color?

Kim: Twenty-four.

Andrea: Twenty-four. So what is the fraction of the yellow and the pink if you're using all 24?… Okay, where do we put that?

Kim: At the top.

Andrea appeared uninterested in hearing about interpretations that the students might be developing, and there was no discussion of the quantities that any of the fractions might represent. When Andrea asked at the end of the lesson what the students had learned about fractions, one responded, "The top number is the numerator and that's the one that we chose stuff to put in, and the bottom number is the denominator and that shows all the things that are together." The parts of the symbol seemed to have been learned as isolated from one another; students did not seem to have had a chance to make sense of fractions as quantities. This lesson, which seems to be representative of Andrea's instruction during the latter part of student teaching, illustrates that her focus during mathematics instruction became more directed toward procedure building with the teacher being the ultimate authority on what procedures were to be learned.

In late April, when asked to describe CGI, Andrea stated, "It is more of letting [students] find [the mathematics] through knowing the appropriate or best questions to ask." Yet, although she appeared to believe in the importance of asking questions to determine what students were thinking, her own questions seemed to be quite controlling; she appeared to want students to learn one predetermined way of solving problems. Any changes she wanted to make in their thinking appeared to be those that aligned students' thinking with her own.

DISCUSSION

Preservice Teachers' Beliefs

In general, the preservice teachers in this study appeared to change their beliefs to a more constructivist orientation about the learning of mathematics during their teacher-preparation program. Furthermore, it seems reasonable to conclude that preservice teachers are able to develop views of instruction that are different from telling.

The greatest change in the preservice teachers' beliefs, as measured on the Belief Scale, occurred during the semester in which the mathematics methods course was taught. This phenomenon indicates that dealing explicitly with mathematics pedagogy and a research-based model of children's mathematical understanding may influence preservice teachers' thinking about teaching and learning mathematics. The preservice teachers' beliefs continued to change fairly significantly during the student-teaching semester to reflect greater concern about the mathematical understanding of the students and greater awareness of the need to help students make sense of mathematics. However, the extent to which the preservice teachers changed beliefs guiding their own teaching varied, as exemplified by Helen's and Andrea's instruction.

Helen's Beliefs and Use of CGI Principles

Overall, Helen appeared to believe that children could solve problems without instruction and that the knowledge she gained from listening to children talking about their thinking could help her make informed decisions. Yet, her use of this information in planning instruction was unclear. Unlike most of the preservice teachers in this study, Helen was supervised during her senior year by an on-site teacher educator who was experienced in using CGI. As a result, she had seen CGI principles being incorporated in regular mathematics instruction before she assumed full responsibility for planning and implementing instruction. Also, the on-site teacher educator encouraged Helen to gather information about students' thinking and to use that information to adapt instruction.

During student teaching Helen involved students in various problem-solving activities that extended beyond the basic arithmetic problem types. She also provided students with various challenging problems, and she encouraged them to share their different solution strategies. Thus, Helen established a learning environment that provided her with numerous opportunities to assess students' thinking and understanding during each lesson. She also indicated that she used students' journal entries as a form of assessment.

On the basis of the levels of mathematics instruction defined by Fennema et al. (1996), Helen's mathematics instruction at the conclusion of the preparation program would be categorized as Level 3: "Provides opportunities for children to solve problems and share their thinking. Beginning to elicit and attend to what children share but doesn't use what is shared to make instructional decisions" (p. 412). And on the basis of the Fennema et al. (1996) levels of cognitively guided beliefs, her beliefs about teaching and learning mathematics also would be categorized as Level 3: "Believes that children can solve problems without instruction. Believes only in a limited way that his or her students' thinking should be used to make instructional decisions" (p. 413).

The instruction Helen provided appeared to be consistent with her beliefs. Students spent most of their time during mathematics solving problems and sharing their solution strategies. Usually two to four problems were completed during each lesson, and students were encouraged to look for connections, to reason, and to communicate mathematically. She generally assigned the same problems to the entire class with students working independently, although they could consult with others in their group of four if they wished. Helen also attended to what the children said or wrote in their journals for the purpose of understanding how they solved the problems, but she did not seem to use this knowledge in planning subsequent instruction as would be expected of a Level 4 teacher. As with the Level 3 teachers in Fennema et al.'s (1996) study, "understanding children's thinking appeared to be an end in itself rather than a means by which to plan instruction" (p. 418). Helen did not deviate from her planned lessons on the basis of what students said or did during an activity, and her instructional planning seemed to be directed mainly by curriculum objectives. Indeed, Helen stated during an interview near the end of student teaching that she planned instruction on the basis of the North Carolina Standard Course of Study, without acknowledging any role for her own understanding of the children's thinking. Helen applied some of the principles of CGI (e.g., she encouraged students to create their own problem solutions and she asked high-level questions), but she missed opportunities to follow up on students' thinking; she failed to fully interpret the sense of what the students said in response to her questions.

Andrea's Beliefs and Use of CGI Principles

When Andrea began student teaching, she appeared to believe that children are able to find their own solutions to problems and that their sharing of solution strategies provides helpful information for planning instruction. Although these beliefs seemed fairly stable throughout Andrea's student-teaching experience, as evidenced by her belief scores and journal entries, such beliefs were not evident in her instruction. Indeed, the relationship between her beliefs and instruction seemed to become more divergent while she gained teaching experience. For example, she asked questions at the beginning of the student-teaching semester that appeared to be attempts to challenge students' reasoning skills by getting them to compare solution strategies. However, she abandoned this level of questioning during the last half of the semester. Also, throughout her teaching, she increasingly guided children to accept particular solution strategies that she had identified, independent of what the students were thinking.

In a written evaluation of one of Andrea's lessons, the on-site teacher educator indicated that Andrea needed to "use every opportunity to model and reinforce any new activity being introduced." Further, Andrea was not encouraged by the on-site teacher educator to investigate in detail what children were thinking. Instead, as Andrea indicated in an interview near the end of student teaching, she taught "CGI-type" lessons "only on days when a lesson was going to be videotaped.… The rest of the time [we went] by the textbook." This choice seemed to be encouraged by the on-site teacher educator. Andrea seemed willing to ask students questions and to create an environment in which children could give wrong answers without being embarrassed; she encouraged students to explain their reasoning for problem solutions. We cannot know, of course, whether this is a sufficient base of knowledge and beliefs from which she can progress to develop into a CGI teacher.

Using the levels defined by Fennema et al. (1996), we considered Andrea's beliefs at the conclusion of the preparation program to be in transition between Level 2 (i.e., "Struggling with the beliefs that children can solve problems without instruction and should use their own strategies" [p. 413]) and Level 3. We categorized her level of cognitively guided instruction (Fennema et al., 1996) as Level 2: "Provides limited opportunities for children to engage in problem solving or to share their thinking. Elicits or attends to children's thinking or uses what they share in a very limited way" (p. 412). Andrea occasionally planned lessons that involved problem solving, and during these lessons she had students find and then share their solutions. However, although she often appeared to be listening to and accepting a student's thinking, she did not seem to actively try to understand how the student solved the problem. She did not ask probing questions to gain clarity, and she often ignored answers that were incorrect.

Case Comparisons and Contrasts

It is not clear why these two preservice teachers differed in their use of CGI principles during student teaching, nor is it clear whether either preservice teacher's progress might be representative of the progress preservice teachers, in general, could make. Did having an on-site teacher educator who was an experienced CGI teacher affect Helen's progress? If so, would Andrea have attained instructional Level 3 (Fennema et al., 1996) had her on-site teacher educator been more knowledgeable of CGI? Is becoming a Level 3 teacher a realistic expectation for preservice teachers even though some experienced teachers do not achieve this level after 4 years of CGI experience (Fennema et al., 1996)? Did Helen's more constructivist set of beliefs at the beginning of her preparation program affect the extent to which she was able to use what she had learned about CGI, compared with Andrea's beliefs and level of use? Did the second major (i.e., psychology for Helen and speech and communication for Andrea) affect the background knowledge that each preservice teacher brought to the teaching experience?

It is encouraging that both Helen and Andrea came to believe that children's mathematical thinking is important and that instruction needs to be based on problem solving. At differing points during their full-time student teaching, each demonstrated competencies in encouraging students to find their own solutions to problems, and they asked questions that encouraged critical thinking and reasoning skills. Perhaps this is all that can be expected of preservice teachers; ability to apply knowledge gained from listening to children's solution strategies may not be a realistic expectation for the 2-year sequence of professional course work during an undergraduate teacher preparation program. At the same time that preservice teachers are being prepared in CGI, they also are gaining baseline knowledge about teaching and learning in general and are identifying basic elements such as their own preferred teaching style; these two factors generally have been addressed by experienced teachers prior to learning about CGI. Developing beginning competencies in the area of mathematics education while achieving in-depth knowledge of children's mathematical thinking and applying that to one's instructional planning may be too much to expect of novice teachers. Indeed, attaining instructional Level 2 (Fennema et al., 1996) as a student teacher may be commendable.

CONCLUSIONS

Students in primary grades can develop fragile mathematics knowledge that produces correct answers in some contexts, but the knowledge may not transfer to other contexts. Similarly, preservice teachers can develop fragile knowledge about teaching that in some contexts may produce behavior consistent with CGI principles, but this behavior may not transfer to all teaching contexts. As we found in this study, preservice teachers may acknowledge the tenets of CGI and yet be unable to use them in their teaching, perhaps in part because of their lack of teaching experience. Unlike the inservice teachers in previous CGI studies, the preservice teachers in this study were establishing a knowledge base about children's thinking and learning and were beginning to develop competencies as mathematics teachers at the same time that they were attempting to construct instructional strategies on the basis of what they were learning about their students' understanding.

There is also a concern about the extent to which the preservice teachers' levels of mathematical understanding may have affected their use of CGI principles during mathematics lessons. For example, Andrea's focus in the excerpt from her lesson on fractions may reflect her own lack of understanding about fractions as quantities as well as her expectation for use of a predetermined procedure for writing the fraction. Because we have no data about the fractions instruction that Helen provided during her field experiences, it is unclear how her teaching of this topic might have changed our perception of her use of the CGI framework. Data collected during the mathematics methods course indicated that she viewed the multiplication of fractions as producing larger answers and division of fractions as producing smaller answers. There was no opportunity to observe how this misunderstanding might have affected her instruction during student teaching.

Beliefs related to the use of CGI principles appear to be manifested by each teacher in the ways that instruction is carried out in the classroom. Considerable personal reflection on one's beliefs and behavior would seem to be necessary for one to develop coherent pedagogy; short, reflective journal entries may not provide adequate opportunity for reflection. Other contexts for reflection (e.g., debriefings after classroom observations by an outsider, meetings with peers to discuss the progress of using CGI principles) may be necessary. It is not clear whether preservice teacher education programs can structurally accommodate these needed "reflection events."

The results of this study seem to be counter to the previous research finding that preservice teachers' beliefs are resistant to change (Calderhead & Robson, 1991; Holt-Reynolds, 1992; Kagan, 1992; McDiarmid, 1990; Schram et al., 1988; Zeichner & Liston, 1987). On the basis of their beliefs scores, interview statements, and reflective journal entries, we conclude that, on average, the preservice teachers in this study changed their beliefs to a more constructivist orientation as a result of their teacher preparation program. On the basis of the findings of this study, we believe that CGI may provide preservice teachers with the foundation for an intelligible and plausible alternative approach to teaching and learning mathematics while at the same time offering them opportunities to link their new set of beliefs to previous conceptions (Posner et al., 1982). Using the taxonomy of problem types and solution strategies as a guide for planning instruction, listening to how children solve problems, and exploring children's geometrical thinking may have provided the preservice teachers in this study with the reinforcement needed to support changes in their beliefs.

The results of this study support Kagan's (1992) conclusions that extensive field experiences and linkages between theory and practice are essential elements for changing preservice teachers' beliefs. The belief changes that occurred for the preservice teachers in this study may be attributed, in part, to the numerous opportunities they had to interact with and study students during field experiences (i.e., 10 hours per week of internship across three semesters prior to full-time student teaching). Also, the case studies, assessment interviews, and observational activities that the preservice teachers completed during the mathematics methods course may have influenced belief changes because of the opportunity each provided for connecting the content of the methods course to the exigencies of classroom teaching. Data on Kagan's second factor (i.e., working with classroom teachers engaged in ongoing self-reflection) were not examined as part of this study, although reflection is an expected part of a PDS teacher's role.

Overall, the 34 preservice teachers in this study changed beliefs and perceptions about mathematics instruction across the 2-year sequence of professional course work during their undergraduate program. We cannot be certain, however, whether the changes were fundamental or superficial. We also cannot be certain of the effects that different factors had on their changing beliefs. The data indicate the possibility that intensity of experience and a focus on children's thinking in the mathematics methods course may be keys for helping preservice teachers change their views. Programs of minimal duration and programs that provide limited field experiences and give minimal attention to focusing on the needs of children may not be as successful in facilitating changes in preservice teachers' perceptions. Preparation in the use of CGI principles through a 5-week module during the mathematics methods course seemed to affect preservice teachers' beliefs about teaching and learning mathematics, and their beliefs continued to change significantly during student teaching. Yet, findings from the more in-depth study of Helen and Andrea indicate that the extent to which they are able to incorporate these beliefs in their instruction varies. Perhaps the amount of time (i.e., five sessions) spent on CGI during the mathematics methods course may have been insufficient for these novice teachers. Another factor, however, appears to be the amount of consistency that exists among the philosophical perspectives of the teacher educators with whom preservice teachers work. Helen and Andrea completed the same mathematics methods course, completed student teaching at the same school and at the same grade level, followed the same school mathematics curriculum, and were supervised by the same university teacher educators. The major difference in their student-teaching experiences was that Helen's on-site teacher educator was an experienced CGI teacher and Andrea's on-site teacher educator had limited knowledge of CGI. Thus, although we believe that providing preservice teachers with a robust research-based model of children's thinking during a mathematics methods course changes their beliefs about teaching and learning mathematics, their abilities to incorporate these beliefs during student teaching may depend on the support preservice teachers receive from the classroom teacher who supervises their student-teaching experiences. In Helen's case, the mathematics methods course, university teacher educators, and on-site teacher educators held consistent philosophical perspectives. Andrea did not experience this level of coherence and thus may have been placed in the awkward position of believing in one approach to teaching and learning mathematics and having to follow a different approach because of the environment to which she was assigned. It appears that if preservice teachers are to internalize coherent applications to teaching and learning mathematics, the environment in which they student teach and the support they receive need to be consistent with the principles being advocated in their professional preparation program.

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