SOUTH DAKOTA SOUTH DAKOTA SOUTH DAKOTA
Counts Counts Counts

1. Play the strategy game "Take Two"; Place seven chips in a row. Two players take turns removing one or two chips each turn. The person to remove the last chip is the winner. Is "Take Two" a fair game? In a fair game, each player has an equal chance of winning. This game can be played with chips, two-colored counters, checkers, cubes, buttons, pennies, squares of colored paper, etc. Although the students may need help understanding the problem and the constraints under which they are to work, avoid giving too much guidance. View this task as more than an exercise for which students are seeking a correct answer. Ask your students to make and test conjectures. Ask them to record their conjectures as they play the game more times. A nice whole-group exploration would be to collect the results of games played by pairs around the room and to see if there are trends. A variety of ways of representing the data will help the class determine whether they think the game is fair. In an alternative version of the game, the winner forces the opponent to remove the last chip.

Does it make a difference who plays first? At what point in the game can you tell who will win the game? Is there one of you who is winning more than the other? What can that person share about his or her strategy in playing? How would the game change if players used eight or nine chips?

2. The Noslo twins, Tanya and Travis received a birthday cake from their Aunt Colleen. Aunt Colleen is a mathematics teacher, so she made the cake in a special shape, a 2 x 3 rectangle, with markings like a geoboard. The twins decided to make a drawing of the cake on a geoboard and use one rubber ban too cut the cake into exactly two pieces so that they would each get a fair share. In how many different ways can they divide the cake?

What do you think they mean by "fair share"? Is there more than one way to cur the cake? Can you categorize the types of cuts you're making? How do you know your cuts create fair shares? Can you come up with any extensions to the problem?

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3. A teacher was cleaning for a rummage sale. In one container he found over 500 each of letters A, B, and C. He decided to use these letters to create a three-letter code name for each person in his class, adults as well as students. Are there enough different combinations so that all of the students and adults will have their own three-letter code name?

Materials needed: numerous cards containing letters A, B, and C

paper

What is meant by different combinations?

Do you see any patterns?

How can you tell if you have all the possible combinations?

Explain how you figured out a strategy for solving the problem.

4. How many rectangles appear in the figure below?

Is a square a rectangle?

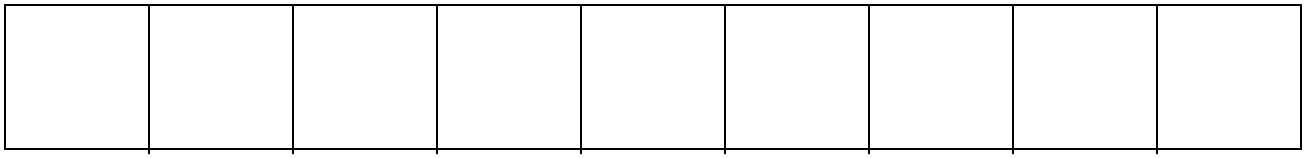
Are all the rectangles the same size?

Have you found any hidden rectangles?

Can rectangles be overlapping?

Do you see a pattern in how many of the different sized rectangles there are?

Is there a way you can use a pattern to sum the different-sized rectangles?



5. South Main School has 618 students. Each student will have his or her picture taken next week by a photographer hired by the school. The photographer uses rolls of film having twenty-four exposures each. how many rolls of film will the photographer have to buy?

Can you create a model to show how the problem can be solved?

Are there any other ways you could represent the problem?

Were there any "left-over's?"
What does the left-over part mean?

How did others around you represent or solve the problem?

6. Working with a hundreds chart, how many numbers can be covered that contain *only* the digit 1? If we used the digits 1 and 2, how many numbers could be covered that contain only the digits 1 or 2 or both? By following this pattern, how many digits are needed before the hundreds chart is half covered?

What conjectures do you have about when we'll reach 1/2?

Is there anything unclear about the problem? If so talk with others to see if you can clarify it.

What patterns are emerging?

7. The owner of a greeting card store wanted to decorate the front window with pictures of snowmen. Her design was finished when she decided to add color to the snowmen. She wanted to use four colors: one color for each of the hat, head, middle section, and bottom section of the snowmen. In how many different ways can the snowmen be colored for the window display?

How many snowmen do you think you'll be able to make?

Can you organize the data?

Is there a way to pattern the change in color from one snowman to the next?

8. The Attibute family was going for a picnic. They made sandwiches from items that were in their refrigerator. They did not count all the sandwiches but they did keep tract of the various sandwiches they made. From the following information, can you help them determine how many sandwiches were made? A sandwich does not have more than one piece of any ingredient, although it may contain several different ingredients.

13 sandwiches had a slice of cheese.

14 had a slice of salami

13 had a slice of tomato.

8 had a slice of cheese and slide of tomato.

3 had only a slice of salami.

5 had a slice of tomato, a slice of cheese, and a slice of salami.

8 had a slice of tomato and a slice of salami.

Can you tell how many types of sandwich are possible?

What strategies are you using to determine what goes on the sandwiches?

Can you think of any ways to organize the information in order to keep track of it?

Which pieces of information tell you exactly what was on a sandwich? Which don't?

9. If you have twelve students in a row, how many children must be "moved" so that the line of twelve alternates between boys and girls in this situation we will count a move when one person is taken out of line and inserted in another place. Other students may glide to the side to allow room for inserting a person in a new position. This glide does not count as a move.

Can you find a general pattern for the number of moves for any number of students in a line that has the same number of boys and girls and that has the girls originally at one end and the boys at the other end?

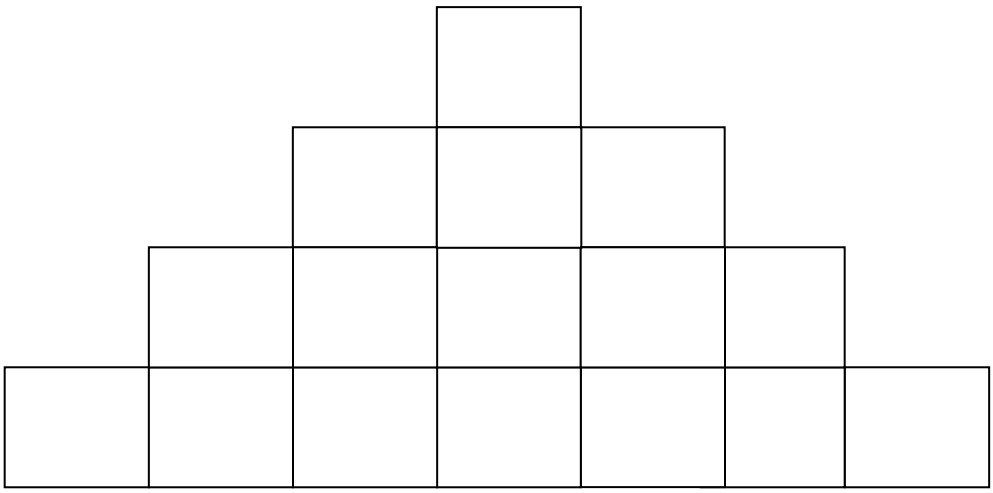
What is a move?

Can you develop any "moving strategies?

Can you create a model to show how the children are moving?

Do you see a pattern to describe how the students are moved?

10. The steps below are made of squares. How many can you count? What if we add another row at the bottom? How many squares could you count? Continue adding rows along the bottom. Can you find a pattern that describes how the total number of squares is growing?



Are all the squares the same size?

Do you see any patterns?

Have you found any hidden squares?

11. To encourage people to attend a special concert, radio station MATH decides to give out 1242 free tickets. The station decides to distribute tickets in the following manner. Each day the station will give away half of the tickets in its possession. How many days will it take before MATH has only one ticket to give?

Estimate when you think all but one ticket will be given out.

Can you think of any ways to organize the data in order to help you think about the problem?

Do you see a pattern?

12. Can you decide what numbers should go in each of the boxes that does not contain a number? Is there a best choice for the number that would be placed in the box?

Another choice for this problem is the one below.

It doesn't become necessary for students to worry about satisfying the pattern in two directions until they are missing enough information that they need to use both rules in order to place a missing number in the box.

What clues helped you select a number?

How do you know that you've chosen a number that works here?

Are there any other numbers that you could put here?

Could this number show up anywhere else in the pattern?

Is it possible to add rows or columns to this pattern?

What other patterns do you see?

Can you create a pattern that goes in more than one direction for the other students to solve?

Can you project what numbers should be put in place of the*s?

13. Eric loved to count. One day Ms. Fox dumped some cubes on Eric's desk. As might be expected, Eric began to count the cubes. He reported the following to Ms. Fox. "When I count the cubes by three, I have one left over; when I count by four, I have one left over; and when I count by seven, I have none left over." From this information can Ms. Fox determine how many cubes Eric has?

What are the clues in the problem that help you?

What can you tell about the number from what Eric said?

Is your solution the only possible one?

14. Herta and Vivienne were excited on their first day of school, especially when their teachers gave them some colored cubes and told them to make up a question to explore. Even though they only had two different colors of cubes, purple and gold. Herta and Vivienne started making towers. First they make towers that were two cubes tall, them three cubes tall and four cubes tall. They began to wonder just how many different towers they could build using only those two colors.

How many towers can you build that are two cubes tall when you have cubes of only two colors to use? How can you be sure that you have made all of the towers? Continue to work with only tow colors of cubes. How many different towers can you build that are three cubes tall? Four cubes tall? How can you be sure that you have made all the towers? What patterns do you notice?

Are you finding any patterns for making your towers?

How do you know if you have all the towers?

What relationships can you see by studying the towers you've already built?

Are there any strategies you can use to help you generate towers?

15. Brownies to Share

Predict: Imagine that you have seven brownies to share equally among four friends. About how many brownies do you think each person will get?

Task: Find out exactly how many brownies each person will get. Be sure that each person gets exactly the same share.

16. Big Macs cost $1.59 each and you have $10.00.

With calculators available, consider and answer the following questions. In each case, show your work and explain how you arrived at your answer.

- **Can you afford to buy 10? Why or why not?**
- **How many can you afford to buy? Did you remember to include tax?**
- **At what sales tax rate can you afford one additional Big Mac?**
- **Explain how you arrived at your answer.**
- **Create two additional questions that arise from this situation and this data.**

17. Given: 92-37=?

Write a word problem, with realistic data, that would require someone to find this difference.

For the different problems that students read:

- **What is a reasonable estimate for your problem?**
- **How did you get your estimate?**
- **What is the actual answer to your problem?**
- **Explain how you arrived at your answer.**

18. At a department store sale, you are buying a $50 sweater that you selected from a table that says "25% OFF. You also have a coupon for an additional 10% off on any purchase.

Sunday, December 23, 10-5

Take an additional

10% OFF EVERYTHING

in the store*

***For example**

Regular price:	**$60.00**
Less the original 25% discount:	**$45.00**
Less an additional 10% discount:	**$39.00**

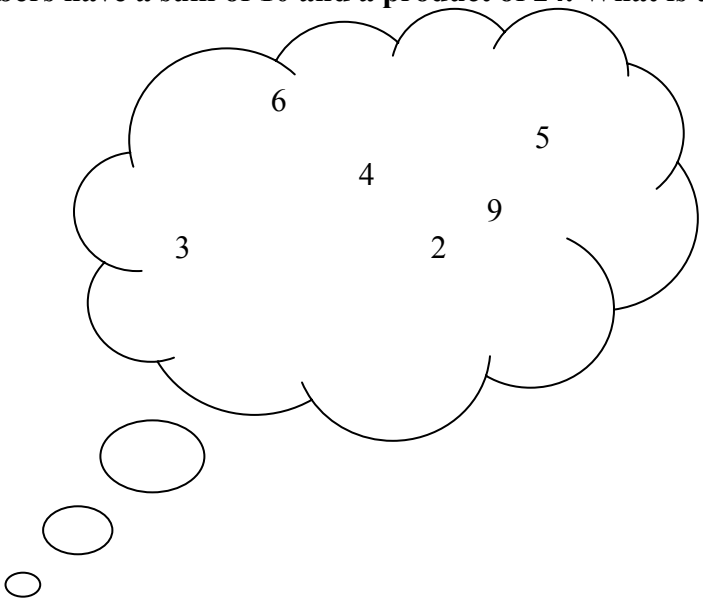
The cashier takes 25% off the original price and then takes an additional 10% off. She asks you for $33.75. Write what you would explain to the cashier to justify why this price is not as good as the bargain in the coupon.

19. You class does some research and finds out that

- **Hot dogs come in packages of 8 hot dogs for $2.50**
- **Hot dog rolls come in packages of 6 for $0.90 and 12 for $1.50**
- **Hamburgers come in packages of 8 patties for $4.00**
- **Hamburger buns come in packages of 8 for $1.00 and 12 for $1.65**

You expect that 24 students and 6 adults will come to the picnic you are planning. You also expect that most but not all students will have dog and most of the adults will have a hamburger. Decide how many packages of each you should buy, find the cost of the food, explain why you made your decisions, and show how you arrived at the cost.

20. Tow numbers have a sum of 10 and a product of 24. What is their difference?



21. **Mark must travel to work and back by bus five days a week. His choices of bus fares are listed below. Which one is his best buy?**

Bus Fares	
One-Way Ticket	$1.60
One-Day Pass (unlimited rides)	$3.00
Booklet (5 round-trip tickets)	$14.00
One-Week Pass (unlimited rides)	$20.00

22. **The cafeteria workers served 928 people on Monday and twice that many on Tuesday. Wednesday they served half the total number of people that were served on Monday and Tuesday. How many people were served during the three days?**

23. **The price of a painting has been increasing $1.25 every three months. Its price is now $60. What was the price of the painting nine months ago?**

24. **There are six parks in my town. Each of the parks is connected to the other parks by a bicycle path. How many bicycle paths connect the parks?**

25. **Mrs. Ellis has 36 students in her class. She gave a fitness test to two thirds of the students in her class and three fourths of them passed. How many students didn't pass the fitness test?**

26. **I am a four digit number. My ones digit is four less than my hundreds digit, which is three times my tens digit, which is half my thousands digit, which can be divided by 1, 2, and 3 with out a remainder. What number am I?**

27. **Six boxes of pencils arrived at the school store. Seven students each bought a dozen pencils. Six pencils remained. How many pencils were originally in each box?**

28. **Eric built a square fence. Each side of the fence was 6 meters long and there was a fence post in each corner. There were more fence posts every two meters along the fence. How many fence posts were there in all?**

29. **At the finish line of the race, Sue was six meters behind Gwen. Lynn was seven meters ahead of Ivan, who was two meters ahead of Gwen, Charles was five meters behind Sue, and Peter was one meter ahead of Charles. Who won the race?**

30. **Shelly paid a total of $5.95 for four packages of noodles and a can of tuna fish. Each of the packages of noodles cost $1.29. How much did the can of tuna fish cost?**

31. **Denise has six coins. Half of her coins are quarters. One third of her coins are nickels. One sixth of her coins are dimes. How much money does she have?**

32. **The parking meter requires .25 for each half hour. Martha parked her car at 11:35 a.m. and expects to return to her car at 2:00 p.m. What are the fewest number of quarters she should put into the meter to avoid getting a parking ticket?**

33. **There are 24 cans of soup in a case. Each can of soup weights 250 grams. How many kilograms does a case of soup weigh, not counting the box?**

34. **I have eight coins. One fourth of the coins are dimes, half of the coins are quarters, and the rest of the coins are nickels. How much money do I have?**

35. **Mike built a garage for his tractor. His garage is 8 feet longer and 10 feet wider than his tractor, which is 20 feet long and 6 feet wide. What is the floor area of his new garage?**

36. **Notebooks sell 5 for $3.00. How many notebooks did I buy if I spent $30 on notebooks?**

37. **If you add together the digit in the millions place, the digit in the ten thousands place, and the digit in the ones place and divide that answer by eight, what is the quotient?**

8,964,312